

Lesson 1.10: LinesThe Slope of a Line

The slope m of a nonvertical line that passes through the points $A(x_1, y_1)$ and $B(x_2, y_2)$ is:

$$m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

Practice: Find the slope of the line between the two points given.

1. $P(2,1)$ and $Q(23,25)$

$$m = \frac{(25-1)}{(23-2)} = \frac{24}{21}$$

2. $A(1, -3)$ and $B(-5, 2)$

$$m = \frac{(2 - (-3))}{(-5 - 1)} = -\frac{5}{6}$$

Point Slope Form of an Equation of a Line

An equation of the line that passes through the point (x_1, y_1) and has a slope m is:

$$y - y_1 = m(x - x_1)$$

Practice: Find the equation of the line in each of the problems below. Leave the equation in point slope form.

1. Through point $(1, -3)$ with slope $-\frac{3}{4}$.

$$y - (-3) = -\frac{3}{4}(x - 1)$$

$$y + 3 = -\frac{3}{4}(x - 1)$$

2. Through points $(4, -3)$ & $(-4, 3)$.

$$m = \frac{(3 - (-3))}{(-4 - 4)} = -\frac{6}{8} = -\frac{3}{4}$$

$$y - (-3) = -\frac{3}{4}(x - 4)$$

$$y + 3 = -\frac{3}{4}(x - 4)$$

Slope Intercept Form of the Equation of a Line

An equation of the line that has slope m and has y -intercept b is:

$$y = mx + b$$

Practice: Find the equation of the line in each of the problems below. Leave the equation in slope-intercept form.

1. Find the equation of a line with slope -2 through the point $(1,3)$.

$$\begin{aligned} y &= -2x + b \\ 3 &= -2(1) + b \\ 3 &= -2 + b \\ +2 \quad +2 & \\ b &= 5 \end{aligned}$$
$$y = -2x + 5$$

2. Find the equation of a line through points $(0,4)$ and $(-2,10)$

$$\begin{aligned} m &= \frac{(10-4)}{(-2-0)} = \frac{6}{-2} = -3 \\ b &= 4 \end{aligned}$$
$$y = -3x + 4$$

Vertical and Horizontal Lines

An equation of the vertical line through (a, b) is $x = a$.

An equation of the horizontal line through (a, b) is $y = b$.

Parallel and Perpendicular Lines

Two non-vertical lines are parallel if and only if they have the same slope.

A slope that is perpendicular to another slope is its opposite (sign) reciprocal.

Practice:

1. Find the equation of a line perpendicular to $y = \frac{-1}{2}x + 3$ through the point $(1, -2)$.

$$\begin{aligned} m &= 2 \\ (1) &= 2(-2) + b \\ 1 &= -4 + b \\ +4 \quad +4 & \\ \hline b &= 5 \end{aligned}$$

$$y = 2x + 5$$

2. Find the equation of a line parallel to $6x - 2 = 2y$ through the point $(0,5)$.

$$\begin{aligned} \frac{6x - 2}{2} &= \frac{2y}{2} & m &= 3 \\ y &= 3x - 1 & b &= 5 \end{aligned}$$

$$y = 3x + 5$$