Assignment 10-10

Find the radius of convergence.

1. $\sum_{n=0}^{\infty} \frac{(2x)^n}{n+1}$

2. $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n^2}$

Find the interval of convergence. Remember to check endpoints when using the Ratio Test.

3. $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n}$

4. $\sum_{n=0}^{\infty} \left(\frac{x}{3}\right)^n$

5. $\sum_{n=1}^{\infty} \frac{(-1)^n (x+3)^n}{n(n+1)}$

6. $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}$

7. $\sum_{n=0}^{\infty} (2n)! x^{2n}$

8. $\sum_{n=0}^{\infty} \frac{x^n}{4^{n+1}}$

9. $\sum_{n=1}^{\infty} \frac{(-1)^n (x-3)^n}{n3^n}$

10. $\sum_{n=0}^{\infty} \frac{(-1)^{n+1} x^n}{n+1}$

11. $\sum_{n=0}^{\infty} \frac{(-1)^n (x-5)^n}{2^{n+1}}$

12. $\sum_{n=1}^{\infty} \frac{(-1)^n x^{2n}}{2n}$

13. $\sum_{n=0}^{\infty} \frac{(n+1)! x^n}{n!}$

14. $\sum_{n=1}^{\infty} \frac{n+1}{n} (6x)^n$

15. Given $f(x) = \frac{1}{2!} + \frac{x}{3!} + \frac{x^2}{4!} + \frac{x^3}{5!} + \cdots + \frac{x^n}{(n+2)!} + \cdots$

a. For what values of x does the given series converge? Show work.

b. Let $g(x) = x^2 f(x)$. Write the Maclaurin series for $g(x)$, showing the first four terms and the general term.

c. Write $g(x)$ in terms of an elementary function.

d. Write $f(x)$ in terms of the same elementary function.

16. Let f be the function given by $f(x) = e^{-x^2}$.

a. Find the first four terms and the general term of the power series for $f(x)$ about $x = 0$.

b. Find the interval of convergence of this power series for $f(x)$.

c. Use the first four nonzero terms of $f(x)$ to approximate $f(0.8)$.

d. Show this approximation has an error less than 0.007.

17. Let f be the function given by $f(t) = \frac{3}{1+t^2}$.

a. Find the first four nonzero terms and the general term for the power series expansion of $f(t)$ about $t = 0$.

b. Given $g(x) = \int_0^x f(t)\,dt$, find the first four nonzero terms and the general term for the power series expansion of $g(x)$ about $x = 0$.

c. Find the interval of convergence of the power series for $g(x)$.