

Assignment 10-3

Find a geometric power series for each of the following functions. Show four terms and the general term. Also give the series using sigma notation and give the interval of convergence.

1. $\frac{1}{1-3x}$

2. $\frac{2}{1-x^3}$

3. $\frac{x}{1+x}$

4. $\frac{1}{1+(-x-3)}$

5. $\frac{3}{4x}$

6. $\frac{1}{2-2x}$

Find a function for each of the following geometric power series. Also give the interval of convergence.

7. $\sum_{n=0}^{\infty} (2x)^n$

8. $\sum_{n=0}^{\infty} \left(-\frac{1}{2}\right)^n x^n$

9. $\sum_{n=0}^{\infty} 4(x-1)^n$

10. $\sum_{n=1}^{\infty} (x^2)^n$

11. $\sum_{n=0}^{\infty} (\sin x)^n$

12. Use the result of the example on your notes page to write a power series for $f(x) = e^{x^2}$. Show four terms and the general term.

13. Use the result of the example on your notes page to write a power series for $g(x) = xe^x$. Show four terms and the general term.

14. Find a geometric power series for $g(x) = \frac{1}{1+x}$. Show four terms and the general term.

15. Use the answer to Problem 14 to find a power series for $\frac{1}{1+x^2}$.

16. Use integration to find a power series for $\arctan x$.

Use the function $f(x) = \sum_{n=1}^{\infty} \frac{x^n}{n}$ to find the following. Answer using sigma notation.

17. $f(-x)$

18. $f'(x)$

19. $\int_0^t f(x) dx$

Use the function $g(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots + \frac{(-1)^n x^{2n}}{(2n)!} + \dots$ to find the following.

Show four terms and the general term.

20. $g(\sqrt{x})$

21. $g'(x)$

22. $\int_0^x g(t) dt$

23. Use the power series for $f(x) = \ln(x)$ from the example in your notes to find a simplified answer for $f(1)$.

Determine whether the following series converge or diverge. Find the sum when possible.

24. $\sum_{n=0}^{\infty} \left(\frac{e}{\pi}\right)^n$

25. $\sum_{n=0}^{\infty} \left(\frac{\pi}{e}\right)^n$

26. $\sum_{n=0}^{\infty} \frac{2n+1}{n+1}$

27. $\sum_{n=2}^{\infty} \frac{n!}{(n-2)!}$