Assignment 10-4

1. Use the definition to find a fourth degree Maclaurin Polynomial for $f(x)=\frac{1}{e^x}$.
2. Use the polynomial from Problem 1 to approximate $\frac{1}{\sqrt{e}}$.
3. Use the definition to find a fourth degree Maclaurin Polynomial for $f(x)=e^{3x}$.
4. Use the definition to find a fifth degree Maclaurin Polynomial for $g(x)=xe^x$.
5. Use the definition to find a third degree Taylor Polynomial centered at $c=1$ for $f(x)=\sqrt[3]{x}$.
6. Use the definition to find a fourth degree Taylor Polynomial centered at $c=1$ for $h(x)=\ln x$.
7. Use the polynomial from Problem 6 to approximate $\ln 1.3$.
8. Use the definition to find a Taylor Series centered at $c=\frac{\pi}{4}$ for $f(x)=\sin x$.
9. Use the definition to find a Taylor Series centered at $c=0$ for $f(x)=\cos(2x)$. Show four terms (Zero terms don't count.) and a general term.
10. Use the definition to write a Maclaurin Series for $f(x)=\frac{1}{1+x}$. Show four terms and the general term.
11. Write a geometric series expansion for $f(x)=\frac{1}{1+x}$. Also give the interval of convergence.
12. Write four terms and the general term of the Taylor series expansion of $f(x)=\frac{1}{x-1}$ about $x=2$.
13. Use the series from Problem 12 to find four terms and the general term of the series expansion about $x=2$ for $\ln|x-1|$.
14. The Taylor Series of a function about $x=3$ is given by
$f(x)=1+\frac{3(x-3)}{1!}+\frac{5(x-3)^2}{2!}+\frac{7(x-3)^3}{3!}+\cdots+\frac{(2n+1)(x-3)^n}{n!}+\cdots$ What is the value of $f'''(3)$?
15. Let $f(x)$ be a function such that $f(0)=2$, $f'(x)=3f(x)$, and the n^{th} derivative of f is given by $f^{(n)}(x)=3f^{(n-1)}(x)$.
    (a) Give the first four terms and the general term of the Taylor Series for f centered at $x=0$.
    (b) Find $f(x)$ by solving the differential equation $f'(x)=3f(x)$ (that is $y'=3y$) with the initial condition $f(0)=2$.
16. Let f be the function defined by the power series $f(x)=2+2x+2x^2+2x^3+\cdots+2x^n+\cdots$
If $g'(x)=f(x)$ and $g(0)=2$, then $g(x)=?$ Show four terms and the general term.
17. The Taylor series for a function f about $x=0$ is $2+\frac{4}{3}x+\frac{8}{9}x^2+\frac{16}{27}x^3+\cdots+\frac{2^{n+1}x^n}{3^n}+\cdots$ for $-1<x<1$. Calculator Allowed.
    (a) Write the first four nonzero terms and the general term for f', the derivative of f.
    (b) Using the appropriate second-degree Taylor polynomial approximate $f(0.2)$ and $f'(0.2)$.
    (c) Use the values found in part (b) to approximate the equation of the tangent line to f at $x=0.2$.