Assignment 10-5

Use an elementary series to give a series for each of the following functions. Show four terms and *the general term.*

1. $f(x)=e^{-4x}$ 2. $g(x)=\cos(3x)$ 3. $f(x)=2\sin x^2$ 4. $h(x)=(x-1)\ln x$

Write each of the following series as a function using elementary functions.

5. $x-\frac{x^3}{2!}+\frac{x^5}{4!}-\frac{x^7}{6!}+\cdots$

6. $1-x^2+\frac{x^4}{2!}-\frac{x^6}{3!}+\frac{x^8}{4!}-\cdots$

7. $\frac{x-1}{x}-\frac{(x-1)^2}{2x}+\frac{(x-1)^3}{3x}-\frac{(x-1)^4}{4x}+\cdots$

8. $1+x-\frac{x^2}{2!}-\frac{x^3}{3!}+\frac{x^4}{4!}+\frac{x^5}{5!}-\cdots$

Find the value of each of the following using an elementary function.

9. $1-\frac{\pi^2}{2!}+\frac{\pi^4}{4!}-\frac{\pi^6}{6!}+\cdots$

10. $(e-1)-\frac{(e-1)^2}{2}+\frac{(e-1)^3}{3}-\frac{(e-1)^4}{4}+\cdots$

11. $5+\frac{4^2}{2!}+\frac{4^3}{3!}+\frac{4^4}{4!}+\cdots$

12. $-\frac{2^3}{3!}+\frac{2^5}{5!}-\frac{2^7}{7!}+\frac{2^9}{9!}-\cdots$

Determine the convergence or divergence of each series. Show justification and name the test used. If possible, find the sum of the series.

13. $\sum_{n=1}^{\infty}\frac{(-1)^n}{n}$ 14. $\sum_{n=0}^{\infty}\frac{(-1)^n\sqrt{n}}{n^2+1}$ 15. $\sum_{n=1}^{\infty}\frac{(-1)^n n^3}{n^3+2}$ 16. $\sum_{n=1}^{\infty}\frac{(-1)^n n}{\ln n}$

17. $\sum_{n=1}^{\infty}\frac{\cos(n\pi)}{n}$ 18. $\sum_{n=1}^{\infty}\left(\frac{3}{4}\right)^n$ 19. $\sum_{n=1}^{\infty}\frac{4n+2}{5n-1}$ 20. $\sum_{n=1}^{\infty}2\pi^{-n}$

21. $\sum_{n=1}^{\infty}\frac{(-1)^n n}{5n^2-1}$ 22. $\sum_{n=0}^{\infty}\frac{(-2)^n}{5^{n+1}}$ 23. $1-\frac{\left(\frac{\pi}{4}\right)^2}{2!}+\frac{\left(\frac{\pi}{4}\right)^4}{4!}-\frac{\left(\frac{\pi}{4}\right)^6}{6!}+\cdots$

Given $f(x)=1-\frac{4}{3}(x-2)^2+\frac{16}{5}(x-2)^4-\frac{2^6}{7}(x-2)^6+\cdots$ is a Taylor Series expansion for $f(x)$ find:

24. a general term for the series.
25. the center of the series.
26. $f(2)$
27. $f'(2)$
28. $f''(2)$
29. $f^{(11)}(2)$
30. $f^{(12)}(2)$
31. $f'(x)$

32. Is the point $(2,1)$ on this same function a local minimum, a local maximum, or neither. Justify.

Write an expression for the *n*th term of these sequences. Assume $n = 1, 2, 3, \ldots$

33. $5, \frac{5}{2}, \frac{5}{6}, \frac{5}{24}, \frac{5}{120}, \cdots$

34. $\frac{1}{4}, \frac{1}{7}, \frac{1}{10}, \frac{1}{13}, \cdots$

Determine if the following sequences converge or diverge.

35. $\frac{5}{4}, \frac{8}{7}, \frac{11}{10}, \frac{14}{13}, \cdots$

36. $a_n=\frac{n^3}{n^2+2}$

37. Find the value of $2-\frac{2}{3}+\frac{2}{9}-\frac{2}{27}+\cdots$

38. Find a third degree Taylor Polynomial for $f(x)=\tan x$ centered at $c=\frac{\pi}{4}$.