Assignment 10-7

Use the Integral Test to show convergence or divergence.

1. $\sum_{n=1}^{\infty} \frac{1}{n+2}$

2. $\sum_{n=1}^{\infty} \frac{1}{e^n}$

3. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}\left(\sqrt{n}+2\right)}$

4. $1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{4}}+\cdots$

5. $\frac{\ln 2}{4}+\frac{\ln 3}{9}+\frac{\ln 4}{16}+\frac{\ln 5}{25}+\cdots$

Explain why the Integral Test cannot be used for these series.

6. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n+2}$

7. $\sum_{n=1}^{\infty} \frac{1}{n-2}$

8. $\sum_{n=1}^{\infty} \frac{n^2}{n+2}$

Use the *p*-series Test to show convergence or divergence.

9. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}}$

10. $1+\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}+\frac{1}{4\sqrt{4}}+\cdots$

11. $\sum_{n=1}^{\infty} \frac{1}{n^e}$

Determine the convergence or divergence by any method.

12. $\sum_{n=1}^{\infty} \left(\frac{1}{e}\right)^{\ln n}$

13. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[4]{n^5}}$

14. $\sum_{n=1}^{\infty} \left(\frac{5}{4}\right)^n$

15. $\sum_{n=1}^{\infty} \frac{2n}{\sqrt{n^2+2}}$

16. $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

17. $\sum_{n=1}^{\infty} \left(\frac{1}{n^2}+\frac{1}{n^3}\right)$

18. $\sum_{n=1}^{\infty} \frac{e^n}{3^{n+1}}$

19. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n}}$

20. $\sum_{n=1}^{\infty} \frac{(-1)^n (n+1)}{\sqrt[3]{n}}$

21. If $f(x)=\frac{1}{1+x}$

a. Write a fourth degree Maclaurin Polynomial. Hint: It is **not** necessary to find any derivatives.

b. Write a power series for the function using Σ notation.

c. Approximate $f(.2)$ using the polynomial from part a.

d. Find the Alternating Series error (remainder) bound for this polynomial.

e. Find the actual value of $f(.2)$.

f. Find the actual error in the approximation from part c.

g. Find the number of terms from the Maclaurin Polynomial needed to approximate $f(.2)$ with an error (remainder) less than .001.

22. Use one of the elementary functions to find the **simplified** value of the following series.

$$(e-1)-\frac{(e-1)^2}{2}+\frac{(e-1)^3}{3}-\frac{(e-1)^4}{4}+\frac{(e-1)^5}{5}+\cdots$$

23. Write the first four nonzero terms and the general term of the Taylor series for $g(x)=\frac{3x^2}{1+x^3}$ about $x=0$.

24. Find the interval of convergence of the series in Problem 23.

25. Use integration and your series from Problem 23 to find four terms of a series for $f(x)=\ln(1+x^3)$.

26. Use your series from Problem 25 to find an approximation for $\ln\frac{9}{8}$ with an error of less than 0.001.