Assignment 10-8

Use the Direct Comparison Test to determine convergence or divergence.

1. $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^2+4}$
2. $\displaystyle\sum_{n=5}^{\infty}\frac{1}{n-4}$
3. $\displaystyle\sum_{n=2}^{\infty}\frac{2}{\sqrt{n}-1}$
4. $\displaystyle\sum_{n=1}^{\infty}\frac{1}{5^n+1}$
5. $\displaystyle\sum_{n=1}^{\infty}\frac{3^{n+1}}{4^n+2}$
6. $\displaystyle\sum_{n=1}^{\infty}\frac{\ln n}{n+2}$
7. $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n!}$

Use the Limit Comparison Test to determine convergence or divergence.

8. $\displaystyle\sum_{n=1}^{\infty}\frac{n}{n^2+4}$
9. $\displaystyle\sum_{n=1}^{\infty}\frac{2}{\sqrt{n^2+2}}$
10. $\displaystyle\sum_{n=1}^{\infty}\frac{2n^2-3n+2}{3n^5+n-4}$
11. $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n\sqrt{n^2+3}}$
12. $\displaystyle\sum_{n=1}^{\infty}\frac{4}{2n+\sqrt{n^2+3}}$
13. $\displaystyle\sum_{n=1}^{\infty}\sin\frac{1}{n}$

14. Use the integral test to determine the convergence of $\displaystyle\sum_{n=1}^{\infty}\frac{n}{\left(n^2+1\right)^2}$.

15. Use the p-series test to determine the convergence of $\displaystyle\sum_{n=1}^{\infty}\frac{\sqrt{n}}{n}$.

Determine the convergence of each of the following by any convergence test.
Do not use the same test twice.

16. $\displaystyle\sum_{n=0}^{\infty}4\left(\frac{1}{6}\right)^n$
17. $\displaystyle\sum_{n=1}^{\infty}\frac{1}{3^n-4}$
18. $\displaystyle\sum_{n=1}^{\infty}\frac{1}{3^n+4}$
19. $\displaystyle\sum_{n=1}^{\infty}\frac{n}{3n+4}$
20. $\displaystyle\sum_{n=2}^{\infty}(-1)^n\frac{5}{n-1}$

21. Given $f(x)=e^{-x}$:
    a. Write a fourth degree Taylor Polynomial centered at $c=1$.
    b. Write a power series for $f(x)$ using Σ notation.
    c. Approximate $f(1.1)$ using the Taylor polynomial from part a.
    d. Find the actual value of $f(1.1)$.
    e. Find the upper limit for the error (remainder) for your approximation in part c.
    f. Find the number of terms from the Taylor Polynomial needed to approximate $f(1.1)$ with an error (remainder) less than .001.

22. Let f be the function given by $f(x)=\sin\left(3x+\frac{\pi}{4}\right)$.
    a. Find $T(x)$ the third-degree Taylor polynomial for f about $x=0$.
    b. Find the coefficient of x^{18} in the Taylor series for f about $x=0$.
    c. Use the Lagrange error bound to show that $\left|f(0.1)-T(0.1)\right|<0.0003$.
    d. Write a third-degree Taylor polynomial for $f'(x)$ about $x=0$.