Assignment 10-9

Use the ratio test to determine convergence or divergence if possible.

1. $\sum_{n=1}^{\infty} \frac{n!}{n^3}$
2. $\sum_{n=0}^{\infty} n\left(\frac{2}{3}\right)^n$
3. $\sum_{n=1}^{\infty} \frac{3^n}{n^3}$
4. $\sum_{n=1}^{\infty} \frac{(-3)^n}{n!}$
5. $\sum_{n=1}^{\infty} \frac{(2n)!}{n3^n}$
6. $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$
7. $\sum_{n=0}^{\infty} \frac{(2n+1)!!}{n!} = \sum_{n=1}^{\infty} \frac{(2n+1)(2n-1)(2n-3)\cdots 3\cdot 1}{n(n-1)(n-2)\cdots 2\cdot 1}$

Determine convergence or divergence using any test.

8. $\sum_{n=1}^{\infty} (-1)^n \frac{3}{n}$
9. $\sum_{n=1}^{\infty} \frac{3}{n}$
10. $\sum_{n=0}^{\infty} \frac{1}{3^n}$
11. $\sum_{n=1}^{\infty} \frac{4}{n\sqrt{n}}$
12. $\sum_{n=1}^{\infty} (-1)^n \frac{3n}{n+1}$
13. $\sum_{n=1}^{\infty} \frac{3n+2}{n^2+2n-4}$
14. $\sum_{n=2}^{\infty} \frac{2^n}{\ln n}$
15. $\sum_{n=1}^{\infty} \frac{|\cos n|}{4^n}$
16. $\sum_{n=1}^{\infty} 4\left(\frac{5^n}{3^{n+1}}\right)$
17. $\sum_{n=3}^{\infty} \frac{(n-2)3^n}{n!}$

18. Which of the following series is/are equivalent to $\sum_{n=1}^{\infty} \frac{2n}{n+1}$?

a. $\sum_{n=0}^{\infty} \frac{2(n+1)}{n+2}$ b. $\sum_{n=0}^{\infty} \frac{2n}{n+1}$ c. $1+\sum_{n=2}^{\infty} \frac{2n}{n+1}$ d. $\sum_{n=1}^{\infty} \left(2-\frac{2}{n+1}\right)$ e. $\frac{7}{3}+\sum_{n=3}^{\infty} \frac{2n}{n+1}$

19. The function f has derivatives of all orders for all real numbers x. Given $f(3)=-2$, $f'(3)=4$, $f''(3)=2$, and $f'''(3)=-9$.
   a. Write the third degree Taylor polynomial for f about $x=3$.
   b. Use your part a answer to approximate $f(2.5)$.
   c. Given $\left|f^{(4)}(x)\right| \le 3$ use the Lagrange error bound on the approximation to $f(2.5)$ found in part b to find a range of possible values for $f(2.5)$.
   d. Write the fourth degree Taylor polynomial for $g(x)=f(x^2+3)$ about $x=0$.
   e. Use your answer to part d to determine if g has a local minimum or a local maximum at $x=0$.

20. Given the series $f(x)=3-15x+75x^2-375x^3+\cdots$ (calculator allowed)
   a. Write an expression for $f(x)$ using sigma notation.
   b. Find the interval of convergence of the geometric series.
   c. Find the exact value of $f(.1)$.
   d. Can the exact value of $f(.3)$ be found? If it can, find it. If not, explain why not.
   e. Approximate $f(.1)$ using the first four terms of the series.
   f. Find the alternating series error bound for this approximation.
   g. Find the actual error for this approximation.