Assignment 2-2

1) $f(x) = (x^2-2)(4x+3)$
$f'(x) = (2x)(4x+3) + (x^2-2)(4)$
$f'(x) = 8x^2 + 6x + 4x^2 - 8$
$f'(x) = 12x^2 + 6x - 8$

2) $f(x) = (x^2-2)(4x+3)$
$= 4x^3 + 3x^2 - 8x - 6$
$f'(x) = 12x^2 + 6x - 8$

3) $f(x) = \frac{2x+1}{x^2+2}$
$f'(x) = \frac{(x^2+2)(2) - (2x+1)(2x)}{(x^2+2)^2}$
$= \frac{2x^2+4-4x^2-2x}{(x^2+2)^2}$
$= \frac{-2x^2-2x+4}{(x^2+2)^2}$

4) $f(x) = \frac{x^2-4}{x+2} = \frac{(x-2)\cancel{(x+2)}}{\cancel{(x+2)}} = x-2$
$f'(x) = 1$ * Not diff. @ $x=-2$

5) $g(x) = \frac{2}{5x^2} = \frac{2}{5}x^{-2}$
$g'(x) = -\frac{4}{5}x^{-3}$
$= -\frac{4}{5x^3}$

6) $y = \frac{12x^2-4}{4} = 3x^2-1$
$y' = 6x$

7) $f(t) = \frac{1}{t^2}(t^3-t^2) = t^{-2}(t^3-t^2)$
$= t^0(t-1)$
$= t-1$
$f'(t) = 1$

8) $g(x) = 2(x^2+5x-3)$
$g'(x) = 2(2x+5)$
$= 4x+10$

9) $f(x) = \frac{2x-3}{3x-2}$
$f'(x) = \frac{(3x-2)(2)-(2x-3)(3)}{(3x-2)^2}$
$f'(x) = \frac{5}{(3x-2)^2}$

10) $y = x^2\sin(x)$
$y' = (2x)\sin(x) + (x^2)(\cos(x))$
$y' = 2x\sin(x) + x^2\cos(x)$

11) $y = \sqrt{x}(x+1) = x^{3/2} + x^{1/2}$
$y' = \frac{3}{2}x^{1/2} + \frac{1}{2}x^{-1/2}$

12) $f(x) = \frac{x^2-c}{x^2+c}$
$f'(x) = \frac{(x^2+c)(2x)-(x^2-c)(2x)}{(x^2+c)^2}$
$= \frac{2x^3+2cx-2x^3+2cx}{(x^2+c)^2}$
$= \frac{4cx}{(x^2+c)^2}$

13) $f(x) = \frac{\cos(x)}{x^3}$
$f'(x) = \frac{(x^3)(-\sin(x)) - [\cos(x)(3x^2)]}{(x^3)^2}$
$= \frac{-x^3\sin(x) - 3x^2\cos(x)}{x^6}$

14) $g(x) = \frac{2x-4}{2\sqrt{x}} = x^{1/2} - 2x^{-1/2}$
$g'(x) = \frac{1}{2}x^{-1/2} + x^{-3/2}$

15) $y = \frac{2(1-\sin(x))}{3\cos(x)} = \frac{2}{3}(\sec(x) - \tan(x))$
$y' = \frac{2}{3}(\sec(x)\tan(x) - \sec^2(x))$

16) $g(x) = x\cos(x)$
$g'(x) = \cos(x) + x(-\sin(x))$
$g'(x) = \cos(x) - x\sin(x)$
$g'(\frac{\pi}{4}) = \cos(\frac{\pi}{4}) - \frac{\pi}{4}\sin(\frac{\pi}{4})$
$g'(\frac{\pi}{4}) = \frac{\sqrt{2}}{2} - \frac{\pi}{4}(\frac{\sqrt{2}}{2})$
$g'(\frac{\pi}{4}) = \frac{4\sqrt{2}}{8} - \frac{\pi\sqrt{2}}{8}$
$g'(\frac{\pi}{4}) = \frac{(4-\pi)\sqrt{2}}{8}$

17) $f(x) = x^2 + 5x - \tan(x)$
$f'(x) = 2x + 5 - \sec^2(x)$
$f'(1) = 2(1) + 5 - \sec^2(1)$
$f'(1) = 7 - \sec^2(1)$
$f'(1) \approx 3.574$

18) $f(x) = \frac{x^2-9}{x-2}$

$f'(x) = \frac{(x-2)(2x) - (x^2-9)(1)}{(x-2)^2}$

$f'(x) = \frac{2x^2-4x-x^2+9}{(x-2)^2}$

$f'(x) = \frac{x^2-4x+9}{(x-2)^2}$

$f'(1) = \frac{(1)^2-4(1)+9}{(1-2)^2}$

$= \frac{1-4+9}{(-1)^2}$

$= 6$

19) $h(t) = \frac{\sec(t)}{t^2}$

$h'(t) = \frac{t^2(\sec(t)\tan(t)) - (\sec(t))(2t)}{(t^2)^2}$

$= \frac{t^2\sec(t)\tan(t) - 2t\sec(t)}{t^4}$

$= \frac{t\sec(t)[t\tan(t)-2]}{t^4}$

$= \frac{\sec(t)[t\tan(t)-2]}{t^3}$

$h'(\pi) = \frac{\sec(\pi)[\pi\tan(\pi)-2]}{\pi^3}$

$h'(\pi) = \frac{-1[\pi(0)-2]}{\pi^3}$

$h'(\pi) = \frac{2}{\pi^3}$

20) $y = \frac{4x^3}{3} = \frac{4}{3}x^3$

$y' = 4x^2$

$y'' = 8x$

21) $f(x) = \frac{x}{x+1}$

$f'(x) = \frac{(x+1) - x}{(x+1)^2}$

$f'(x) = \frac{1}{(x-1)^2}$

$f'(-2) = \frac{1}{(-2-1)^2} = \frac{1}{(-3)^2} = \frac{1}{9}$

$y - 2 = \frac{1}{9}(x+2)$

22) $g(x) = (2x-1)(x^2+3)$

$g'(x) = 2(x^2+3) + (2x-1)(2x)$

$g'(x) = 2x^2+6+4x^2-2x$

$g'(x) = 6x^2-2x+6$

$6 = 6x^2-2x+6$

$-6 \qquad\qquad -6$

$6x^2-2x = 0$

$2x(3x-1) = 0$

$x = 0$ OR $x = 1/3$

23) $f(x) = \frac{x^2}{x+1}$

$f'(x) = \frac{(x+1)(2x) - (x^2)}{(x+1)^2}$

$f'(x) = \frac{x^2+2x}{(x+1)^2} = \frac{x(x+2)}{(x+1)^2}$

$x(x+2) = 0$

$x = 0$ OR $x = -2$

24) $f(x) = \frac{x}{\sin(x)}$

$f(\frac{\pi}{6}) = \frac{\pi}{6} \cdot \frac{1}{\sin(\pi/6)} = \frac{\pi}{6} \cdot \frac{1}{1/2} = \frac{\pi}{3}$

$f(\frac{\pi}{2}) = \frac{\pi}{2} \cdot \frac{1}{\sin(\pi/2)} = \frac{\pi}{2}(1) = \frac{\pi}{2}$

$AROC = \frac{f(\frac{\pi}{2}) - f(\frac{\pi}{6})}{\frac{\pi}{2} - \frac{\pi}{6}}$

$= \frac{\frac{\pi}{2} - \frac{\pi}{3}}{\frac{\pi}{2} - \frac{\pi}{6}}$

$= \frac{\frac{3\pi}{6} - \frac{2\pi}{6}}{\frac{3\pi}{6} - \frac{\pi}{6}}$

$= \frac{\pi}{6} \cdot \frac{6}{2\pi}$

$= \frac{1}{2}$

25) $f(x) = \frac{x}{\sin(x)}$

$f'(x) = \frac{\sin(x) - x(\cos(x))}{[\sin(x)]^2}$

$f'(\frac{\pi}{6}) = \frac{\sin(\frac{\pi}{6}) - (\frac{\pi}{6}\cos(\frac{\pi}{6}))}{[\sin(\frac{\pi}{6})]^2}$

$f'(\frac{\pi}{6}) = \frac{\frac{1}{2} - (\frac{\pi}{6} \cdot \frac{\sqrt{3}}{2})}{(1/2)^2}$

$f'(\frac{\pi}{6}) = 4(\frac{6}{12} - \frac{\pi\sqrt{3}}{12})$

$f'(\frac{\pi}{6}) = \frac{6}{3} - \frac{\pi\sqrt{3}}{3}$

$f'(\frac{\pi}{6}) = \frac{6-\pi\sqrt{3}}{3}$

26) $g(x) = x \cdot f(x)$

$g'(x) = x \cdot f'(x) + f(x)$

$g'(2) = 2 \cdot f'(2) + f(2)$

$g'(2) = 2(-2) + 3$

$g'(2) = -4 + 3$

$g'(2) = -1$

27) $h(x) = \frac{f(x)}{x^2}$

$h'(x) = \frac{x^2 f'(x) - f(x)(2x)}{(x^2)^2}$

$h'(2) = \frac{2^2 f'(2) - f(2)(2(2))}{(2^2)^2}$

$h'(2) = \frac{4(-2) - 3(4)}{16}$

$= \frac{-8 - 12}{16}$

$= -\frac{20}{16}$

$= -5/4$

28) $f(x) = \frac{g(x)}{h(x)}$

$f'(x) = \frac{h(x) \cdot g'(x) - g(x) \cdot h'(x)}{[h(x)]^2}$

$f'(3) = \frac{h(3) \cdot g'(3) - g(3) \cdot h'(3)}{[h(3)]^2}$

$f'(3) = \frac{(4)(3) - 2(5)}{[4]^2}$

$= \frac{12 - 10}{16}$

$= 2/16$

$= 1/8$