Assignment 3-2

1. $f(x)=x^5-1$. Let $g(x)=f^{-1}(x)$.
   a. Find $g(x)$.
   b. Graph $f(x)$ and $g(x)$ in the same coordinate plane.
   c. Find $f'(x)$ and $g'(x)$.
   d. Find $f'(1)$ and $g'(0)$. (The points $(1, 0)$ on f and $(0, 1)$ on g are image points).
   e. What is the relationship between the slopes in Part d?

For Problems 2 and 3, let $g(x)$ be the inverse of $f(x)$. Repeat the five parts in Problem 1 for the given function $f(x)$ and the given point on f. (Use the given point on f when finding f'. You will need to use the image point when finding g'.)

2. $f(x)=3x-1$
   Point : $(1,2)$

3. $f(x)=\sqrt{x+1}$
   Point: $(3,2)$

4. Let f and g be inverse functions such that: $\begin{cases} f(-1)=0,\ f(0)=1, \text{ and } f(1)=3 \\ f'(-1)=\frac{4}{3},\ f'(0)=\frac{1}{5}, \text{ and } f'(1)=2 \end{cases}$

   Find each of the following (if possible).

   a. $g'(-1)$ b. $g'(0)$ c. $g'(1)$ d. $g'(2)$ e. $g'(3)$

5. If $f(2)=3$ and $f'(2)=4$, find $\left(f^{-1}\right)'(3)$.

6. If $(1, 2)$ is a point on $f(x)=x^3+2x-1$, find $\left(f^{-1}\right)'(2)$.

7. If $f(x)=x^3-\frac{4}{x}$ $(x>0)$, find $\left(f^{-1}\right)'(6)$.

f and g are inverse functions in Problems 8-10. Find g' at the given value.

8. $f(2)=5$
   $f'(2)=\frac{-2}{3}$
   $g'(5)=$

9. $f(x)=x^5+2x^3-1$
   $f(1)=2$
   $g'(2)=$

10. $f(x)=e^{x-2}$
    $g'(1)=$

For Problems 11-13, use $f'(x)$ to find the largest interval on which $f(x)$ has an inverse function.

11. $f(x)=x^2-3x+10$
    (Interval contains $x=1$)

12. $f(x)=12x-x^3$
    (Interval contains $x=3$)

13. $f(x)=x^3-3x^2+3x$

Differentiate in Problems 14-17.

14. $y=x^5+\frac{1}{x}$
    $\frac{d^2y}{dx^2}=$

15. $\frac{d}{dx}\left(6^{2x}-3\right)^4=$

16. $f(x)=\frac{x^2}{e^x}$
    $f'(x)=$

17. $g(t)=(2t-1)^5$
    $g''(t)=$