

CURVE SKETCHING PRACTICE

Curve Sketching

1. Give the domain (watch for denominator restrictions, radical restrictions).
2. Reduce $f(x)$. Oftentimes, you must factor before you reduce.
3. Find vertical asymptotes (denominator restrictions after reducing) and holes.
4. Give x and y intercepts.
5. Find the end behavior (horizontal asymptotes or other) using highest degree terms of the numerator and the denominator.
6. Find a starting point. (If needed.)
7. Graph.

Practice:

Use the curve sketching recipe to sketch the following curves.

1. $f(x) = x(x - 1)(x + 2)^2$

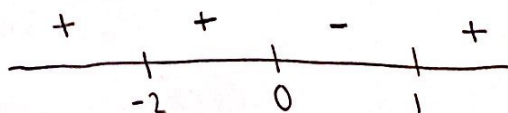
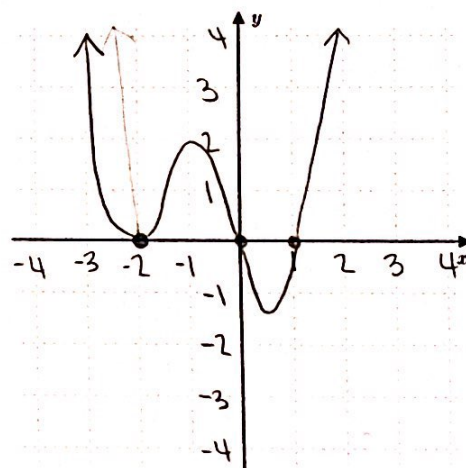
Domain: $x \in (-\infty, \infty)$

y-int: $(0, 0)$

$f(0) = 0(0-1)(0+2)^2 = 0$

x-int: $(0, 0)$ $(1, 0)$ $(-2, 0)$

End Behavior: $\lim_{x \rightarrow \infty} f(x) = \infty$ $\lim_{x \rightarrow -\infty} f(x) = \infty$



$$2. g(x) = \frac{x(x-1)^2(x+3)^3}{x^2(x-1)(x-3)^2}$$

$$\text{Domain: } \{x \in \mathbb{R} : x \neq 0, 1, 3\}$$

$$g_{red}(x) = \frac{(x-1)(x+3)^3}{x(x-3)^2}$$

$$\text{V.A.: } x=0 \quad \& \quad x=3$$

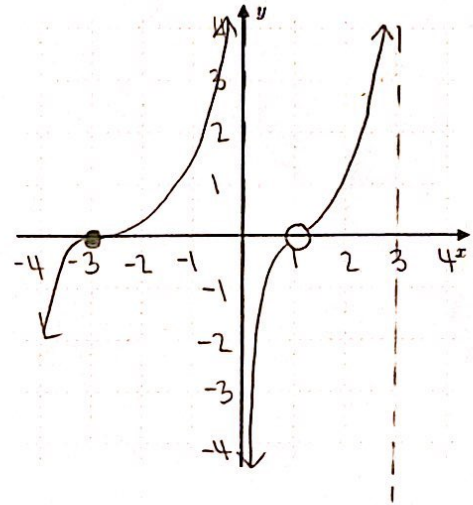
$$\text{Holes: } x=1 \quad (1, 0)$$

$$\text{x-int.: } (-3, 0)$$

$$\text{y-int.: N/A}$$

$$\text{End Behavior: } \lim_{x \rightarrow \infty} g(x) = \infty$$

$$\lim_{x \rightarrow -\infty} g(x) = -\infty$$



$$3. y = \frac{x+1}{\sqrt{x^2-4}}$$

$$\text{Domain: } \{x \in \mathbb{R} : x \notin (-2, 2)\}$$

$$\text{Holes: N/A}$$

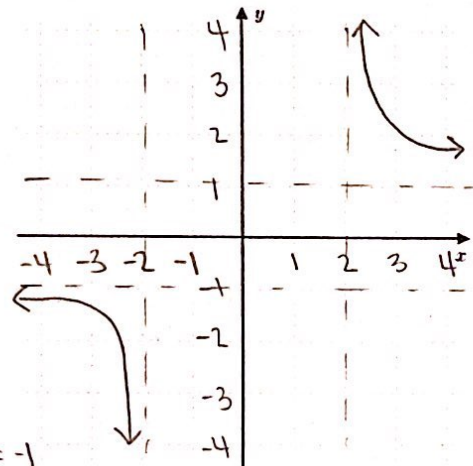
$$\text{V.A.: } x=2 \quad \& \quad x=-2$$

$$\text{x-int.: N/A}$$

$$\text{y-int.: N/A}$$

$$\text{Starting Point: N/A}$$

$$\text{End Behavior: } \lim_{x \rightarrow \infty} \frac{x+1}{\sqrt{x^2-4}} = 1 \quad \lim_{x \rightarrow -\infty} \frac{x+1}{\sqrt{x^2-4}} = -1$$



consider:

$$\lim_{x \rightarrow \infty} \frac{x}{\sqrt{x^2}} = \lim_{x \rightarrow \infty} \frac{x}{|x|} = 1$$

$$\lim_{x \rightarrow -\infty} \frac{x}{|x|} = -1$$