2.8 # 1, 3, 5, 7, 13, 15, 19, 21, $23

1) $f(x) = x^3 + x = x(x^2+1)$
$f(x)$ is continuous on $[1,2]$
$f(1) = (1)^3 + 1 = 2$
$f(2) = (2)^3 + 2 = 10$
$9 \in (f(1), f(2))$
$\Rightarrow f(x) = 9$ for some $c \in (1,2)$

3) $g(t) = t^2 \tan(t)$
$g(t)$ is continuous on $[0, \pi/4]$
$g(0) = 0^2 \tan(0) = 0$
$g(\pi/4) = (\pi/4)^2 \tan(\pi/4) = \frac{\pi^2}{16}$
$\frac{1}{2} \in (g(0), g(\pi/4))$
$\Rightarrow g(t) = \frac{1}{2}$ for some $c \in (0, \pi/4)$

5) $f(x) = x - \cos(x)$
$f(x)$ is continuous on $[0,1]$
$f(0) = 0 - \cos(0) = -1$
$f(1) = 1 - \cos(1)$
$0 \in (f(0), f(1))$
$\Rightarrow f(x) = 0$ for some $c \in (0,1)$
$\Rightarrow x = \cos(x)$ for some value $c \in (0,1)$

7) $\sqrt{c} + \sqrt{c+2} = 3$
Let $f(c) = \sqrt{c} + \sqrt{c+2} - 3$
f is continuous on $[0,2]$
$f(0) = \sqrt{0} + \sqrt{0+2} - 3 = \sqrt{2} - 3$
$f(2) = \sqrt{2} + \sqrt{2+2} - 3 = \sqrt{2} + 1$
$0 \in (f(0), f(2))$
$\Rightarrow f(c) = 0$ for some $c \in (0,2)$
$\Rightarrow \sqrt{c} + \sqrt{c+2} = 3$ for some $c \in (0,2)$

13) $2^x + 3^x = 4^x$
$f(x) = 2^x + 3^x - 4^x$
f is continuous on $[0,3]$
$f(0) = 2^0 + 3^0 - 4^0 = 2 + 1 = 1$
$f(3) = 2^3 + 3^3 - 4^3 = 8 + 27 - 64 = -29$
$0 \in (f(3), f(0))$
$\Rightarrow f(x) = 0$ for some $c \in (0,3)$
$\Rightarrow 2^x + 3^x = 4^x$ for some $c \in (0,3)$

15) $2^x + \frac{1}{x} = -4$
$f(x) = 2^x + \frac{1}{x} + 4$
f is continuous on $[-1, -\frac{1}{5}]$
$f(-1) = 2^{-1} + \frac{1}{(-1)} + 4 = \frac{1}{2} - 1 + 4$
$= \frac{1}{2} + 3$
$= \frac{7}{2}$
$f(-\frac{1}{5}) = 2^{-1/5} + \frac{1}{(-1/5)} + 4$
$= \frac{1}{\sqrt[5]{2}} - 5 + 4$
$= \frac{1}{\sqrt[5]{2}} - 1 < 0$
$0 \in (f(-1), f(-1/5))$
$\Rightarrow f(x) = 0$ for some $c \in (-1, -\frac{1}{5})$
$\Rightarrow 2^x + \frac{1}{x} = -4$ has a solution $c \in (-1, -1/5)$

19) $f(x) = x^7 + 3x - 10$
f is continuous on $[\frac{5}{4}, \frac{3}{2}]$
$f(5/4) = (5/4)^7 + 3(5/4) - 10 < 0$
$f(3/2) = (3/2)^7 + 3(3/2) - 10 > 0$
$0 \in (f(5/4), f(3/2))$
$\Rightarrow f(x) = 0$ (has a root) for some $c \in (\frac{5}{4}, \frac{3}{2})$