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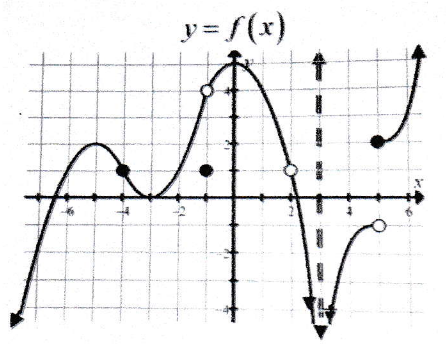
Lesson 1.1: Limits & Continuity

Limits

Informally, a **limit** is a y-value which a function approaches as x approaches some value.

$\lim_{x \to c} f(x) = L$ means as x approaches c, $f(x)$ approaches the y-value of L.

Example 1:



Limits	***Function Values***	***One-Sided Limits***
1. $\lim_{x \to 4} f(x) \approx -1.5$	1. $f(-4) = 1$	1. $\lim_{x \to 5^-} f(x) = -1$
2. $\lim_{x \to -1} f(x) = 4$	2. $f(-1) = 1$	
3. $\lim_{x \to 2} f(x) = 1$	3. $f(2) =$ und.	2. $\lim_{x \to 5^+} f(x) = 2$
4. $\lim_{x \to 3} f(x) = -\infty$ DNE	4. $f(3) =$ und.	
5. $\lim_{x \to 5} f(x) =$ DNE	5. $f(5) = 2$	

Continuity

Informally, a function is continuous where it can be drawn without lifting a pencil.

Roughly, continuous means "connected".

Formally, a function is **continuous** where its limit and function value are the same.

The Three Types of Discontinuities We Will be Working With

1. holes
2. jumps (breaks)
3. vertical asymptotes

*A fourth type of discontinuity is oscillating discontinuity (these rarely appear). (Graph $y = sin\left(\frac{1}{x}\right)$)

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All **discontinuities** can be classified as either removable or non-removable.

Removable discontinuity occur when the function has a limit (holes in the graph).

Non-removable discontinuity occur when the limit of the function does not exist (jumps, vertical asymptotes, or oscillations).

Example 2: List the x-values of the discontinuities of the function $y = f(x)$ graphed in Example 1. State whether or not the discontinuity is removeable.

$x = -1$ (removable) $x = 3$ (non-removable)

$x = 2$ (removable) $x = 5$ (non-removable)

At x-values where a function is **continuous**, limits can be found by direct substitution.

Example 3:

a. $\lim_{x \to 3}(3x^2 + 2) =$ $3(3)^2 + 2 = 3(9) + 2 = 27 + 2 = 29$

b. $\lim_{x \to 1} \frac{x^2+x}{x+1} =$ $\frac{(1)^2+1}{(1)+1} = \frac{2}{2} = 1$

For piecewise functions, one sided limit evaluation is often necessary.

Example 4:

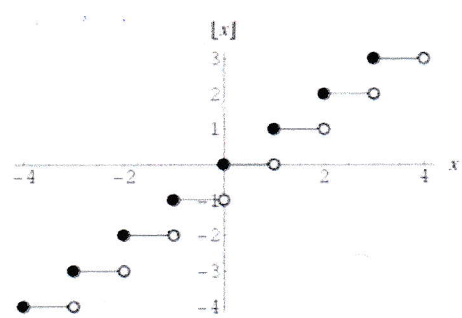
If $f(x) = \begin{cases} 4 - x, & x \le 1 \\ 4x - x^2, & x > 1 \end{cases}$ $\lim_{x \to 1} f(x) =$ 3 $\leftarrow$ b/c $\lim_{x \to 1^+} f(x) = \lim_{x \to 1^-} f(x)$

$\lim_{x \to 1^+} f(x) = 4(1) - (1)^2 = 4 - 1 = 3$ $\lim_{x \to 1^-} f(x) = 4 - (1) = 3$

Another function requiring one- sided limit analysis is a step function called the **Greatest Integer Function** also known as the **Floor Function**.

$$f(x) = \lfloor x \rfloor = the\ greatest\ integer\ less\ than\ or\ equal\ to\ x$$

Example 5:



a. $\lim_{x \to \frac{1}{2}} \lfloor x \rfloor =$ 0

b. $\lim_{x \to 1} \lfloor x \rfloor =$ DNE b/c $\lim_{x \to 1^+} \lfloor x \rfloor \neq \lim_{x \to 1^-} \lfloor x \rfloor$

c. $\lim_{x \to 5^-} \lfloor x \rfloor =$ 4