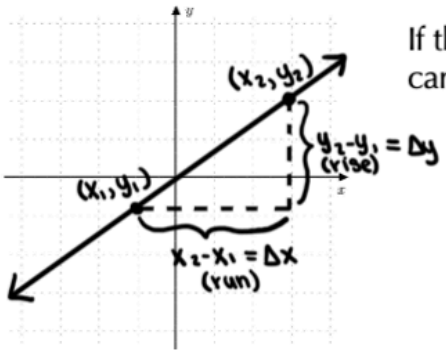
LESSON 1.10: LINES

SLOPE OF A *line*

The slope of a line is essentially the steepness of that line.

The slope describes how quickly a line is increasing or decreasing.



If the points (x_1, y_1) and (x_2, y_2) are on a line, we can find the slope by using the following formula:

$$\text{Slope} = m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Example: Find the slope of the line that goes through the points (2,3) and $(-1,10)$.

$$\text{Slope} = \frac{(10)-(3)}{(-1)-(2)} = \frac{7}{-3}$$

$$m = -\frac{7}{3}$$

POINT SLOPE *form*

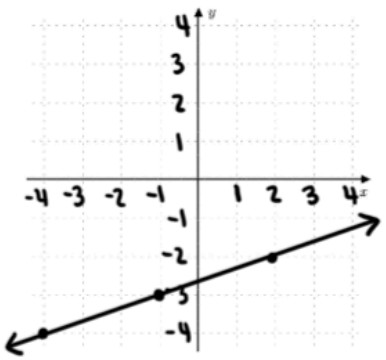
If we know one point (x_1, y_1) on a line as well as the line's slope (m), we can write the equation of a line in point-slope form:

$$y - y_1 = m(x - x_1)$$

Example: Find the equation of the line that goes through the point $(-1, -3)$ and has a slope of $\frac{1}{3}$. Then, graph the line on the axes provided.

$$y - (-3) = \tfrac{1}{3}(x - (-1))$$

$$y + 3 = \tfrac{1}{3}(x + 1)$$



SLOPE INTERCEPT *form*

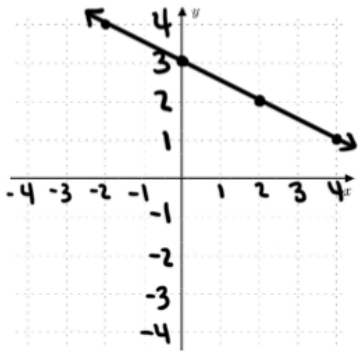
An equation of the line that has slope **m** and y-intercept **b** is:

$$y = mx + b$$

Examples:

1. Write the equation of a line with a slope of $-\frac{1}{2}$ and a y-intercept of 3. Then, graph the equation of the line on the axes provided.

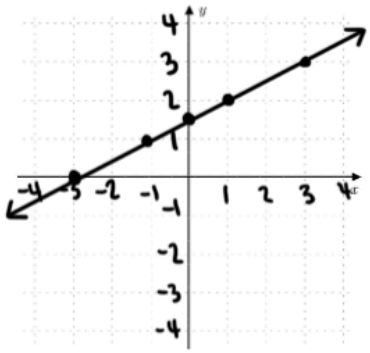
$$y = -\frac{1}{2}x + 3$$



2. Write the equation of a line that goes through the points (1,2) and (−1,1) in slope-intercept form. Then, graph the equation of the line on the axes provided.

$$m = \frac{(1)-(2)}{(-1)-(1)} = \frac{-1}{-2} = \frac{1}{2}$$

$$y = \frac{1}{2}x + b$$
$$1 = \frac{1}{2}(-1) + b$$
$$1 = -\frac{1}{2} + b$$
$$\frac{3}{2} = b$$
$$y = \frac{1}{2}x + \frac{3}{2}$$



VERTICAL AND HORIZONTAL *lines*

An equation of a vertical line through the point (a, b) is $x = a$.

- The slope of a vertical line is undefined.

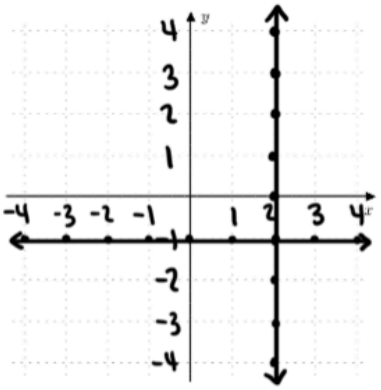
An equation of a horizontal line through the point (a, b) is $y = b$.

- The slope of a horizontal line is 0.

Examples:

1. Graph a vertical line that passes through the point (2,4). $x = 2$

2. Graph a horizontal line that passes through the point (3, −1). $y = -1$



GENERAL EQUATION OF A line

The general equation of a line is:

$$Ax + By + C = 0 \quad (A \neq 0 \text{ or } B \neq 0)$$

We can use algebra to manipulate the general form of a line to put it into slope-intercept form.

Example: What is the slope and y-intercept of the line $2x + 3y - 5 = 0$?

$2x - 3y - 5 = 0$
$+5 \quad +5$
$2x - 3y = 5$
$-2x \quad -2x$
$\frac{-3y}{-3} = \frac{-2x+5}{-3}$
$y = \frac{2}{3}x - \frac{5}{3}$

slope: $\frac{2}{3}$
y-intercept: $(0, -\frac{5}{3})$

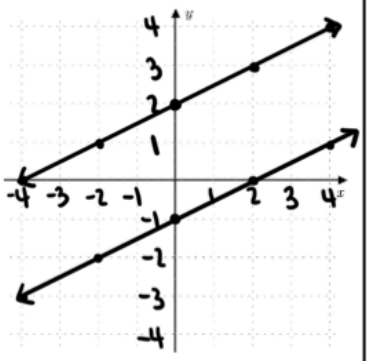
PARALLEL lines

Two lines are parallel if they never intersect and have the same slope.

Example: Are the lines $y = \frac{1}{2}x + 2$ and $2y - x + 2 = 0$ parallel? Graph the lines and explain your reasoning.

$2y - x + 2 = 0$
$-2 \quad -2$
$2y - x = -2$
$+x \quad +x$
$\frac{2y}{2} = \frac{x-2}{2}$
$y = \frac{1}{2}x - 1$

The lines are parallel because they both have a slope of $\frac{1}{2}$.



PERPENDICULAR lines

Two lines are perpendicular if they intersect at a 90° angle.

If the slope of one line is a, then the slope of a line perpendicular to it would be $-\frac{1}{a}$.

- The slopes are opposite (sign) and reciprocal of one another.

Example: Are the lines $y = -\frac{1}{3}x - 3$ and $3y - x + 12 = 0$ perpendicular? Graph the lines and explain your reasoning.

$3y - x + 12 = 0$
$-12 \quad -12$
$3y - x = -12$
$+x \quad +x$
$\frac{3y}{3} = \frac{x-12}{3}$

$y = \frac{1}{3}x - 4$
$m = \frac{1}{3}$

While the signs are opposite, the slopes are not reciprocal of one another, so the lines are not perpendicular. The slope would need to be 3.