ENTRY #1

# Lesson 1.1: Real Numbers

## NUMBER systems

| Name | Symbol | Description | Examples |
|---|---|---|---|
| Natural Numbers | $\mathbb{N}$ | All positive integers. | $1, 2, 3, 4, \ldots$ |
| Integers | $\mathbb{Z}$ | Consists of natural numbers and their opposites (negatives) as well as 0. | $-3, -2, -1, 0, 1, 2, \ldots$ |
| Rational Numbers | $\mathbb{Q}$ | Any number that can be written as a ratio of two integers. $\frac{a}{b}$ *where a and b are integers and* $b \neq 0$. | $\frac{1}{4}, -\frac{2}{3}, 4, \frac{-7}{2}, \ldots$ |
| Irrational Numbers | $\mathbb{R}-\mathbb{Q}$ | Any number that cannot be expressed as a ratio of two integers. Will have an infinite non-repeating list of numbers to the right of the decimal point. | $\sqrt{3}, e, \pi, \sqrt{2}, \ldots$ |
| Real Numbers | $\mathbb{R}$ | Any number that can be represented on a number line line. This includes the set of natural numbers, integers, rational numbers and irrational numbers. | $57, e, \pi, \frac{3}{4}, -\frac{5}{7}, -1, 0, 5, \sqrt{2}, \ldots$ |

## SETS and SET NOTATION

A set is the collection of objects, these objects are called elements of the set.

**Example:** S = {1, 2, 3, 4, 5} represents set S that contains elements 1,2,3,4 & 5.

The symbol $\epsilon$ means "element of". For example, $1 \epsilon S$ means 1 is an element of set S (as shown above).

We use set builder notation to describe a set of numbers without having to list each element individually.

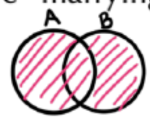
$$A = \{x \underset{\text{"such that"}}{:} \text{"desired characteristic of } x\text{"}\}$$

**Example:** $A = \{x : x \textit{ is an integer and } 0 < x < 7\}$ represents:

0, 1, 2, 3, 4, 5, 6, 7 ← elements in set A

## UNIONS *and* INTERSECTIONS

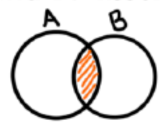
$A \cup B$ means A “ __union__ ” B. A union of two sets consists of all elements in __set A or B or both__ . Essentially, we are “marrying” the two sets without repeating any elements that are in both sets.




**Example:** Let A = {1,2,3,4,5} and B = {1,3,5,7,9}. Find $A \cup B$.

$A \cup B = \{1,2,3,4,5,7,9\}$ *note: 1, 3 & 5 are only stated once.

$A \cap B$ means A “ __intersect__ ” B. An intersection of two sets consists of all elements in __both__ A and B. Essentially, we are listing all elements that the two sets have in common.




**Example:** Let A = {1,2,3,4,5} and B = {1,3,5,7,9}. Find $A \cap B$.

$A \cap B = \{1,3,5\}$

## INTERVAL *notation*

We can also use interval notation to define a __set__ of numbers.

Interval notation is used to describe a set of real numbers between two __endpoints__ .

An __open__ interval does not include endpoints: $(a,b)$


A __closed__ interval does include endpoints: $[a,b]$


You can use a combination of open and closed parenthesis if you want to include one endpoint, but not the other. For example, __$[a,b)$__ or __$(a,b]$__ .

**Examples:** For each set below, draw a number line that would represent the set.

1. (-1,5)


All real #s in between -1 and 5.

2. [2,7]


All real numbers in between 2 and 7 including 2 and 7.

3. (-2,0]


All real numbers in between -2 and 0 including 0.

## "INFINITE" endpoints

We can use interval notation to indicate we want the set of all numbers __greater__ than or __less__ than a particular number.

To do this, we need to use the "infinity symbol" : $\infty$ "positive infinity" $-\infty$ "negative infinity"

__$(-\infty, a)$__ represents all numbers "less than" $a$: 

__$(a, \infty)$__ represents all numbers "greater than" $a$: 

We never use a closed parenthesis with the infinity symbol. $\cancel{[}-\infty, a)$

## ABSOLUTE value

The __absolute__ value of a number $a$ represented by __$|a|$__ , is the distance from $a$ to 0 on the real number line.

We can use set builder notation to define absolute value:

$$|a| = \begin{cases} a : a \geq 0 \\ -a : a < 0 \end{cases}$$

**Examples:** Evaluate.

1. $|2| = 2$
2. $|-3| = 3$
3. $|-e| = e$
4. $|3 - \pi| = -(3-\pi) = -3+\pi$

   $3 < \pi \Rightarrow 3-\pi < 0$

Since absolute value represents a number's distance from 0, we can also use the idea of absolute value to find the distance from one number to another.

If $a$ and $b$ are real numbers, the distance between them on the real number line is:

$$d(a,b) = |b-a|$$

*order does not matter when using absolute value.

**Examples:** Evaluate.

1. $d(3,5) = |5-3| = |2| = 2$
2. $d(-4,7) = |7-(-4)| = |11| = 11$