ENTRY #2

Lesson 1.2: Exponents & Radicals

EXPONENTIAL notation

We can write a product of identical numbers in exponential notation.

If a is any real number and n is a positive integer, then the n^{th} power of a is:

$$a^n = \underbrace{a \cdot a \cdot a \cdot a \cdots a}_{n \text{ times}}$$

The number a is called the base, and n is called the exponent.

Any number (except zero) to the zeroth power is 1, not zero.

We can also have negative exponents: $a^{-n} = \frac{1}{a^n}$

Examples: Evaluate.

1. $\left(\frac{1}{3}\right)^4 = \left(\frac{1}{3}\right)\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)\left(\frac{1}{3}\right) = \frac{1}{81}$

2. $2^0 = 1$

3. $(-3)^{-3} = \left(-\frac{1}{3}\right)^3 = \left(-\frac{1}{3}\right)\left(-\frac{1}{3}\right)\left(-\frac{1}{3}\right) = -\frac{1}{27}$

LAWS OF exponents

Exponent Law	Example	Description
$a^m a^n = a^{m+n}$	$x^3 x^2 = x^{3+2} = x^5$	To multiply two powers of the same base, add the exponents.
$\frac{a^m}{a^n} = a^{m-n}$	$\frac{x^3}{x^2} = x^{3-2} = x$	To divide two powers of the same base, subtract the exponents.
$(a^n)^m = a^{nm}$	$(x^4)^3 = x^{4 \cdot 3} = x^{12}$	To raise an exponent to another power, multiply the exponents.
$(ab)^n = a^n b^n$	$(xy)^4 = x^4 y^4$	To raise a product to a power, raise each factor to that power, then multiply them together.
$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	$\left(\frac{x}{y}\right)^3 = \frac{x^3}{y^3}$	To raise a quotient to a power, raise the dividend and divisor by that power, then divide.
$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$	$\left(\frac{3}{5}\right)^{-n} = \left(\frac{5}{3}\right)^n$	A rational number to a negative power is equivalent to the reciprocal of that rational number raised to the opposite power.

USING LAWS OF *exponents*

Examples: Use the laws of exponents to simplify the expressions below.

1. $a^5a^3 = a^{5+3} = a^8$

2. $\left(\frac{a}{b}\right)^{-3} = \left(\frac{b}{a}\right)^3 = \frac{b^3}{a^3}$

3. $\frac{y^8}{y^2} = y^{8-2} = y^6$

4. $(x^2)^4 = x^{2\cdot 4} = x^8$

5. $(-2y)^3 = (-2)^3(y)^3 = -8y^3$

6. $(c^2c^{-4})^3 = (c^{2-4})^3$
$= (c^{-2})^3$
$= c^{-6} = \frac{1}{c^6}$

7. $\left(\frac{x^2}{y}\right)^{-2}\left(\frac{x^3y}{z^2}\right)^3 = \left(\frac{y}{x^2}\right)^2 \frac{(x^3y)^3}{(z^2)^3}$
$= \frac{y^2}{x^4}\cdot\frac{x^9y^3}{z^6}$
$= \frac{y^{2+3}x^{9-4}}{z^6} = \frac{y^5x^5}{z^6}$

8. $(x^2y^5)^2(2xy^2)^{-3}$
$= x^4y^{10}(2)^{-3}(x)^{-3}(y)^{-6}$
$= x^{4-3}y^{10-6}\left(\frac{1}{2}\right)^3$
$= \frac{1}{8}xy^4$
$= \frac{xy^4}{8}$

RADICALS AND *roots*

The symbol $\sqrt{\ }$ means the "positive square root of".

$\sqrt{a} = b$ means that $b^2 =$ a and $b \geq 0$.

For example, $\sqrt{9} =$ 3 because 3^2 $= 9$.

We can also take the n^{th} root of a number.

If n is any positive integer, then the principle n^{th} root of a is defined as:

$$\sqrt[n]{a} = b \text{ because } b^n = a \quad (\& \ b \geq 0)$$

Examples: Evaluate.

1. $\sqrt[3]{27} = 3$ $(3^3 = 27)$

2. $\sqrt[3]{8} = 2$ $(2^3 = 8)$

3. $\sqrt[3]{-8} = -2$ $((-2)^3 = -8)$

4. $\sqrt[4]{-16}$ $2^4 = 16$ $(-2)^4 = 16$ $\neq -16$
not possible because the exponent is positive

	Property	Example
PROPERTIES OF NTH *roots*	$\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[b]{b}$	$\sqrt[3]{4\cdot 2} = \sqrt[3]{4}\cdot\sqrt[3]{2}$
	$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$	$\sqrt{\frac{5}{9}} = \frac{\sqrt{5}}{\sqrt{9}} = \frac{\sqrt{5}}{3}$
	$\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$	$\sqrt[4]{\sqrt[3]{x}} = \sqrt[4\cdot 3]{x} = \sqrt[12]{x}$
	$\sqrt[n]{a^n} = a$ if n is odd	$\sqrt[3]{4^3} = 4$
	$\sqrt[n]{a^n} = \lvert a\rvert$ if n is even	$\sqrt[4]{2^4} = \lvert 2\rvert = 2$

RATIONAL *exponents*

We can use ___radicals___ to define rational or ___fractional___ exponents.

$$a^{1/n} = \underline{\sqrt[n]{a}}$$

For any rational exponent m/n in lowest terms where m and n are integers and $n \neq 0$:

$$a^{m/n} = \sqrt[n]{a^m}$$

SIMPLIFYING USING ROOT *properties*

Examples: Simplify each expression below.

1. $\sqrt[4]{x^5} = \sqrt[4]{x^4 x} = x(\sqrt[4]{x}) = x^{5/4}$

2. $\sqrt[3]{(27)(8)} = \sqrt[3]{27}\cdot\sqrt[3]{8}$
   $= (3)(2)$
   $= 6$

3. $\sqrt{\frac{9}{2}} = \frac{\sqrt{9}}{\sqrt{2}} = \frac{3}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$ *rationalize the denominator
   $= \frac{3\sqrt{2}}{2}$

4. $\sqrt[4]{3^4} = \lvert 3\rvert = 3$

5. $\sqrt{200} + \sqrt{8} = \sqrt{100\cdot 2} + \sqrt{4\cdot 2}$
   $= 10\sqrt{2} + 2\sqrt{2}$
   $= 12\sqrt{2}$

6. $8^{\frac{2}{3}} = (\sqrt[3]{8})^2$
   $= (2)^2$
   $= 4$