Entry #: 3

Lesson 1.2: Intermediate Value Theorem & Graphing Adjustments

Warm Up:

When an **indeterminate form** $\left(\text{usually } \frac{0}{0}\right)$ is obtained after trying to use direct substitution, it is necessary to use algebraic techniques to take change the form of the limit. Find the following limits.

1. $\lim\limits_{x\to 0} \frac{\cancel{x}}{\cancel{x}(x+1)}$ $\quad \frac{0}{0}$ Form

$= \lim\limits_{x\to 0} \frac{1}{x+1}$

$= \frac{1}{0+1} = 1$

2. $\lim\limits_{x\to 2} \frac{x-2}{x^2-4}$ $\quad \frac{0}{0}$ Form

$= \lim\limits_{x\to 2} \frac{\cancel{(x-2)}}{(x+2)\cancel{(x-2)}}$

$= \lim\limits_{x\to 2} \frac{1}{x+2}$

$= \frac{1}{2+2} = \frac{1}{4}$

3. $\lim\limits_{x\to 1} \frac{x^3-1}{x-1}$ $\quad \frac{0}{0}$ Form

$= \lim\limits_{x\to 1} \frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{(x-1)}}$

$= \lim\limits_{x\to 1} x^2+x+1$

$= 1^2+1+1$

$= 3$

4. $\lim\limits_{x\to 1} \frac{\sqrt{x}-1}{x-1}$ $\quad \frac{0}{0}$ Form

$= \lim\limits_{x\to 1} \frac{(\sqrt{x}-1)(\sqrt{x}+1)}{(x-1)(\sqrt{x}+1)}$

$= \lim\limits_{x\to 1} \frac{\cancel{(x-1)}}{\cancel{(x-1)}(\sqrt{x}+1)}$

$= \lim\limits_{x\to 1} \frac{1}{\sqrt{x}+1} = \frac{1}{\sqrt{1}+1} = \frac{1}{2}$

5. $\lim\limits_{x\to 2^+} \frac{x}{x-2}$ $\quad \frac{0}{0}$ Form

(odd)

V.A @ $x = 2$

x	$f(x)$
2.09	$23.\overline{222}$
2.07	29.57

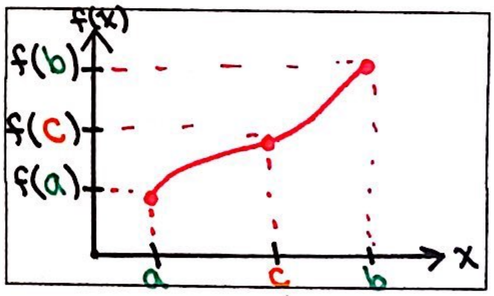
incr.

$\lim\limits_{x\to 2^+} \frac{x}{x-2} = \infty$

(DNE)

Intermediate Value Theorem:

If f is <u>continuous</u> on $[a, b]$ and k is any real y-value between $f(a)$ and $f(b)$, then there is <u>at least one</u> x-value **c** between **a** and **b** such that $f(c) = k$. In other words, f takes on every y-value between $f(a)$ and $f(b)$.



Example: Does the Intermediate Value Theorem guarantee a c-value on the given interval? If so, find the value of c.

a. $f(x) = x^2 - x$

$f(c) = 12, \quad [0,5]$

(1) $f(x)$ is continuous on $[0,5]$ ✓

(2) $f(0) = 0^2 - 0 = 0$

$f(5) = (5)^2 - 5 = 20$

$f(c) \in (0, 20)$ ✓

$\Rightarrow$ IVT guarantees a c-value s.t $f(c)=12$ on the interval $[0,5]$

$x^2 - x = 12$

$-12 \quad -12$

$x^2 - x - 12 = 0$

$(x-4)(x+3) = 0$

$x = 4$ or ~~$x = 3$~~

$\boxed{c = 4}$

b. $g(x) = \frac{x^2-4}{x-2}$

$g(c) = 4, \quad [0,3]$

$g(x) = \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}}$ Hole @ $x = 2$

$\Rightarrow g(x)$ is not continuous on $[0,3]$

$\Rightarrow$ IVT does not guarantee a c-value s.t. $g(c) = 4$ on the interval $[0,3]$

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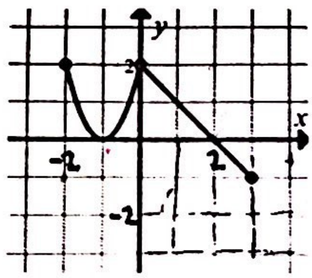
Graphing Adjustments to $y = f(x)$

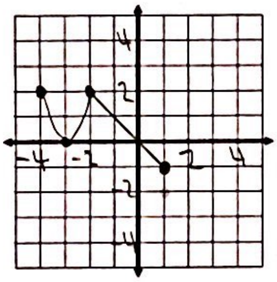
Any adjustment made to a function always produces a function.

Adjustment	Result
$y = -f(x)$	Reflect across the x-axis.
$y = f(-x)$	Reflect across the y-axis.
$y = f(x) + d$	Shift up if $d > 0$, shift down if $d < 0$.
$y = f(x - c)$	Shift right if $c > 0$, shift left $c < 0$.
$y = a \cdot f(x)$	vertical stretch if $a > 1$ vertical squeeze if $0 < a < 1$ *If $a < 1$, a reflection over the x axis is needed
$y = f(bx)$	Horizontal squeeze if $b > 1$, horizontal stretch if $b < 1$ (assuming b is positive, otherwise a reflection is needed).
$y = \lvert f(x) \rvert$	reflect all points below the x axis across the x-axis. Leave points above x-axis alone.
$y = f(\lvert x \rvert)$	completely eliminate all points left of the y-axis. Leave points right of the y-axis alone. Replace the left half of the graph with a reflection of the right half. (y-axis symmetry)

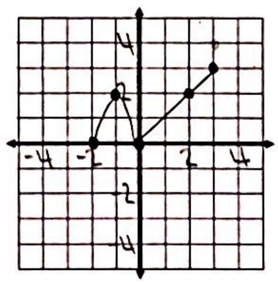
Examples: Use the graph of $y = f(x)$ shown to sketch the following.



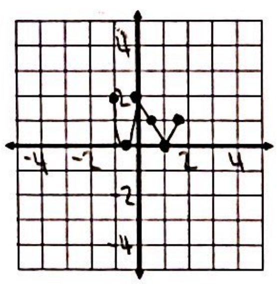
1. $y = f(x + 2)$



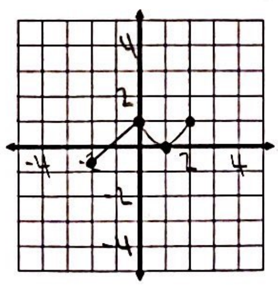
2. $y = -f(x) + 2$



3. $y = \lvert f(2x) \rvert$



4. $y = \frac{1}{2} f(-x)$



5. $y = f(\lvert x \rvert)$