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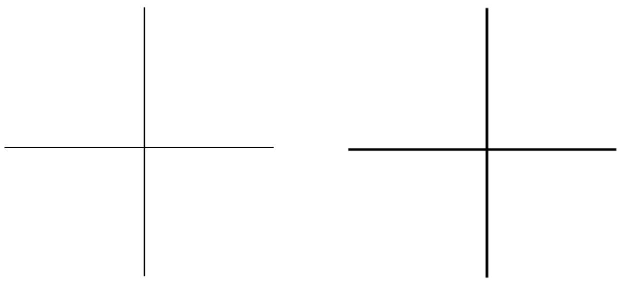
Lesson 1-3: Infinite Limits, Limits at Infinity, & Curve Sketching

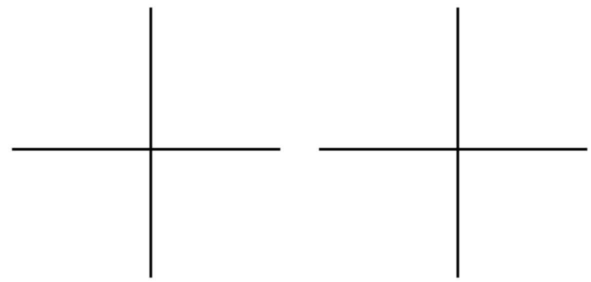
Review:

A ____________ on a graph looks like a hollow circle. It represents the fact that the function approaches the point, but is not actually defined on that precise x-value.

Even Vertical Asymptotes:



Odd Vertical Asymptotes:



Example: Find the hole and any even or odd vertical asymptotes for: $f(x) = \frac{x-1}{(x-1)(x-2)^2(x-4)^3}$.

Infinite Limits:

Previously, we have seen examples where a limit does not exist on a vertical asymptote.

These non-existent limits can be expressed as __________________ if the vertical asymptote is ____________ or if you are finding one-sided limits.

We will write:

Example:

$$\lim_{x\to 2^+} \frac{x+3}{x-2}$$

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Limits at Infinity

If the graph of a function $f(x)$ approaches a _____________________ to the left and/or the right, $f(x)$ is said to have a ___________________ .

If the asymptote is $y = L$, then $\lim_{x\to\infty} f(x) = L$.

In other words, limits at infinity give us ___________________ for graphs of functions.

For "large" values of x, the highest degree terms in the numerator and denominator dominate the other terms and are the only terms you need to consider.

Example: *Note: Make sure you consider highest degree terms not highest degree factors.

$$\lim_{x\to\infty} \frac{(2x+3)(x-1)^2}{(x+2)(3x-1)^2}$$

Rational functions like the one above have at most one horizontal asymptote, so the limit is the same whether x approaches ∞ or $-\infty$.

However ___________________ often have two horizontal asymptotes.

Example:

$\lim_{x\to\infty} \frac{\sqrt{4x^2-3}}{x} =$ 　　　　　　　　　　$\lim_{x\to-\infty} \frac{\sqrt{4x^2-3}}{x} =$

Curve Sketching

1. Give the domain (watch for denominator restrictions, radical restrictions).
2. Reduce $f(x)$. Oftentimes, you must factor before you reduce.
3. Find vertical asymptotes (denominator restrictions after reducing) and holes.
4. Give x and y intercepts.
5. Find the end behavior (horizontal asymptotes or other) using highest degree terms of the numerator and the denominator.
6. Find a starting point. (If needed.)
7. Graph.

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Classwork 1-3: Infinite Limits, Limits at Infinity, & Curve Sketching

Infinite Limits

1. $\lim_{x\to 2^-} \frac{x+3}{x-2} =$

2. $\lim_{x\to 1^+} \frac{x-2}{(x-1)^2} =$

3. $\lim_{x\to 1^-} \frac{x-2}{(x-1)^2} =$

4. $\lim_{x\to 2} \frac{x^2-4}{x-2} =$

5. $\lim_{x\to 2^\pm} \frac{x+1}{x-2} =$

6. $\lim_{x\to 3^\pm} \frac{x^2-3x-1}{x-3} =$

Limits at Infinity

Find the following horizontal asymptotes:

1. $f(x) = \frac{5x^4-3x^2+2}{10x^4+3}$

2. $f(x) = \frac{5x^4-3x^2+2}{10x^5+3}$

3. $f(x) = \frac{5x^4-3x^2+2}{10x^3+3}$

Find the following limits:

1. $\lim_{x\to \infty} \frac{5x^4-3x^2+2}{10x^4+3} =$

2. $\lim_{x\to -\infty} \frac{5x^4-3x^2+2}{10x^5+3} =$

3. $\lim_{x\to \infty} \frac{5x^4-3x^2+2}{10x^3+3} =$

Curve Sketching

Use the curve sketching recipe to sketch the following curves.

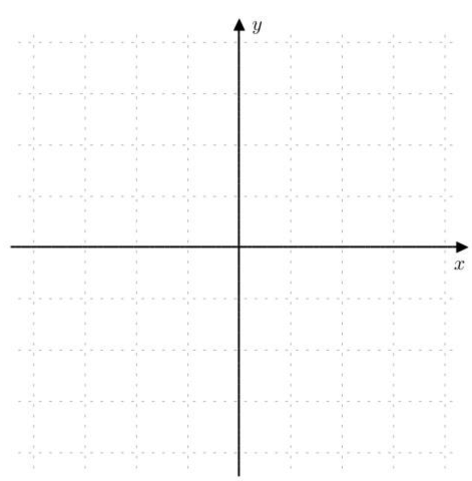
1. $f(x) = x(x-1)(x+2)^2$

   Domain:

   y-int:

   x-int:

   End Behavior:



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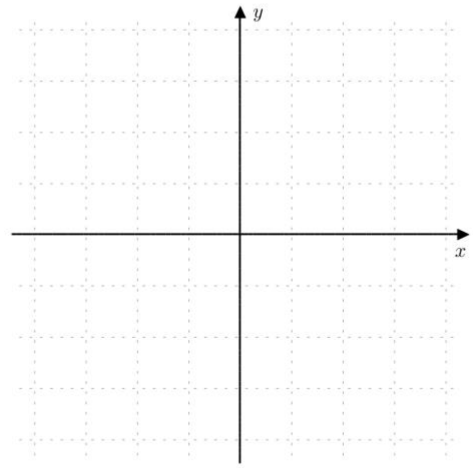
2. $g(x) = \frac{x(x-1)^2(x+3)^3}{x^2(x-1)(x-3)^2}$



Domain:

$g_{red}(x) =$

V.A.:

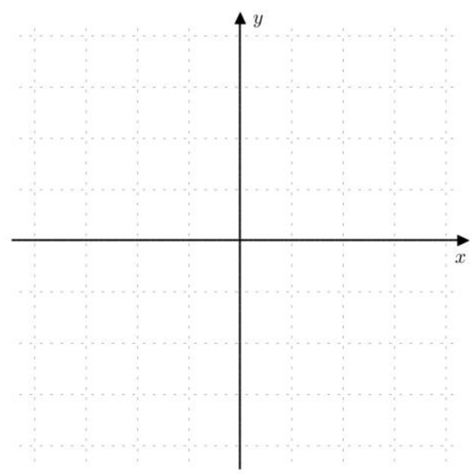
Holes:

x-int.:

y-int.:

End Behavior:

3. $y = \frac{x+1}{\sqrt{x^2-4}}$



Domain:

Holes:

V.A.:

x-int.:

y-int.:

Starting Point:

End Behavior: