Name: Key

Lesson 1.3 & 1.4: Algebraic Expressions & Rational Expressions

Polynomials → General Form: $ax^n + bx^{n-1} + cx^{n-2} + \cdots + lx^1 + mx^0$

constant term

A **polynomial** is an expression of more than two algebraic terms.

Polynomials are defined both by the number of terms they have and the degree of the terms being added.

Example:

Expression	Type	Terms	Degree
$2x^2 - 3x + 4$	trinomial	$2x^2, -3x, 4$	2 (quadratic)
$5x^{10} - 13x$	binomial	$5x^{10}, -13x$	10
6	monomial	$6x^0$	0

Factoring Polynomials

We factor polynomials in order to find their factors.

Factors can be used to find roots of a polynomial. Another name for root is x-intercept (zeros).

Example:

$$\frac{x^{3/2}}{x^{-1/2}} = x^{3/2-(-1/2)} = x^2$$

$$\frac{-3x^{1/2}}{x^{-1/2}} = -3x^{1/2-(-1/2)} = -3x$$

$$3x^{\frac{3}{2}} - 9x^{\frac{1}{2}} + 6x^{-\frac{1}{2}}$$

$$= 3(x^{3/2} - 3x^{1/2} + 2x^{-1/2})$$ ← Factor out the term with the smallest degree

$$= 3x^{-1/2}(\boxed{x^2}\boxed{-3x} + 2)$$

$$= 3x^{-1/2}(x-1)(x-2)$$

Rational Expressions

The domain is all possible x-vales of a given function.

The range is all possible y-values of a given function.

Example: Find the domain and range.

$$f(x) = \frac{x}{x^2 - 5x + 6} = \frac{x}{(x-2)(x-3)} \leftarrow x \neq 2 \text{ OR } 3$$

Domain: $\{x \in \mathbb{R} : x \neq 2 \text{ \& } x \neq 3\}$

OR

$(-\infty, 2) \cup (2, 3) \cup (3, \infty)$

Range: $(-\infty, -5-2\sqrt{6}] \cup [2\sqrt{6}-5, \infty)$ ← see graph

Multiplying and Dividing Rational Expressions

To multiply polynomial expressions, we must use the following property of fractions:

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

To divide polynomial expressions, we must use the following property of fractions:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

Example:

$$\frac{x^2 + 2x - 3}{x^2 + 8x + 16} \cdot \frac{3x + 12}{x - 1}$$

*simplify first

$$= \frac{(x+3)\cancel{(x-1)}}{(x+4)^2} \cdot \frac{(3x+12)}{\cancel{(x-1)}}$$

$$= \frac{(x+3)(3x+12)}{(x+4)^2} = \frac{3x^2 + 21x + 36}{x^2 + 8x + 16}$$

Rationalizing Denominators

The degree of the denominator must be 1

Example 1:

$$\frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

Example 2:

$$\frac{2}{\sqrt[3]{2}} \cdot \frac{(\sqrt[3]{2})^2}{(\sqrt[3]{2})^2} = \frac{2(\sqrt[3]{2})^2}{2} = (\sqrt[3]{2})^2 = 2^{2/3}$$