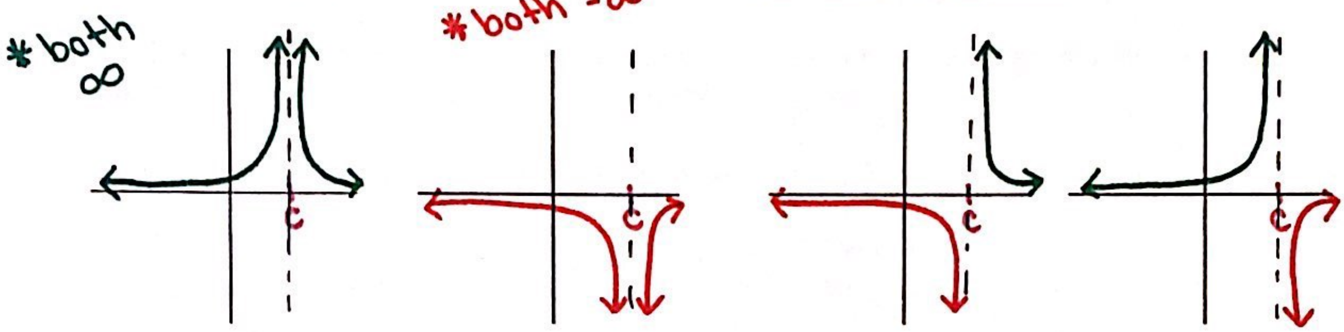
Entry #: 4

Lesson 1.3: Infinite Limits & Limits at Infinity

Review:

A hole on a graph looks like a hollow circle. It represents the fact that the function approaches the point, but is not actually defined on that precise x-value.

Even Vertical Asymptotes: Odd Vertical Asymptotes:



Example: Find the hole and any even or odd vertical asymptotes for: $f(x) = \frac{x-1}{(x-1)(x-2)^2(x-4)^3}$.

$$f(x) = \frac{\cancel{(x-1)}}{\cancel{(x-1)}(x-2)^2(x-4)^3}$$

Hole @ $x=1$

Even V.A @ $x=2$

Odd V.A @ $x=4$

Infinite Limits:

Previously, we have seen examples where a limit does not exist on a vertical asymptote.

These non-existent limits can be expressed as infinite limits if the vertical asymptote is even or if you are finding one-sided limits.

We will write:

$$\lim_{x\to c} f(x) = \infty \quad \text{or} \quad \lim_{x\to c} f(x) = -\infty$$

$\Rightarrow$ the limit does not exist.

Examples:

1. $\lim\limits_{x\to 2^+} \frac{x+3}{x-2}$

odd V.A. @ $x=2$

$\lim\limits_{x\to 2^+} \frac{x+3}{x-2} = \infty$

(DNE)

x	$f(x)$
3	6
2.5	11

increasing as we get closer to 2 from larger values

2. $\lim\limits_{x\to 3} \frac{x^2+1}{x^2-2x-3} = \lim\limits_{x\to 3} \frac{x^2+1}{(x-3)(x+1)}$

odd V.A. @ $x=3$

$\lim\limits_{x\to 3^+} \frac{x^2+1}{x^2-2x-3} \neq \lim\limits_{x\to 3^-} \frac{x^2+1}{x^2-2x-3}$

$\Rightarrow \lim\limits_{x\to 3} \frac{x^2+1}{x^2-2x-3}$ DNE

Limits at Infinity

If the graph of a function $f(x)$ approaches a horizontal asymptote to the left and/or the right, $f(x)$ is said to have a limit at infinity.

If the asymptote is $y = L$, then $\lim_{x\to\infty} f(x) = L$.

In other words, limits at infinity give us end behaviors for graphs of functions.

For "large" values of x, the highest degree terms in the numerator and denominator dominate the other terms and are the only terms you need to consider.

Examples: *Note: Make sure you consider highest degree terms not highest degree factors.

1. $\lim_{x\to\infty} \frac{(2x+3)(x-1)^2}{(x+2)(3x-1)^2} = \frac{2}{9}$

   consider: $\lim_{x\to\infty} \frac{2x^3}{9x^3} = \frac{2}{9}$

2. $\lim_{x\to-\infty} \frac{2x-1}{x-2x^2} = \lim_{x\to-\infty} \frac{2x-1}{-2x^2+x} = 0$

   consider. $\lim_{x\to\infty} \frac{2x}{-2x^2} = \lim_{x\to-\infty} \frac{1}{-x^2} = 0$

3. $\lim_{x\to-\infty} \frac{x^3-1}{x^2+1} = -\infty$

   consider: $\lim_{x\to-\infty} \frac{x^3}{x^2} = \lim_{x\to-\infty} x = -\infty$

Rational functions like the one above have at most one horizontal asymptote, so the limit is the same whether x approaches ∞ or $-\infty$.

However radical functions often have two horizontal asymptotes.

Examples:

1. $\lim_{x\to\infty} \frac{\sqrt{4x^2-3}}{x} = 2$

   consider: $\lim_{x\to\infty} \frac{\sqrt{4x^2}}{x} = \lim_{x\to\infty} \frac{2\sqrt{x^2}}{x}$

   $= \lim_{x\to\infty} \frac{2|x|}{x}$

   $= 2$

2. $\lim_{x\to-\infty} \frac{\sqrt{4x^2-3}}{x} = -2$

   consider: $\lim_{x\to-\infty} \frac{\sqrt{4x^2}}{x} = \lim_{x\to-\infty} \frac{2\sqrt{x^2}}{x}$

   $= \lim_{x\to-\infty} \frac{2|x|}{x}$

   $= -2$

Practice Problems:

Infinite Limits

1. $\lim_{x\to 2^-} \frac{x+3}{x-2} = -\infty$

*odd V.A @ $x=2$

x	$f(x)$
1	4
1.5	-9

decreasing

2. $\lim_{x\to 1^+} \frac{x-2}{(x-1)^2} = -\infty$

* even V.A. @ $x=1$

x	$f(x)$
2	0
1.5	-2

decreasing

3. $\lim_{x\to 1^-} \frac{x-2}{(x-1)^2} = -\infty$

* Even V.A @ $x=1$

x	$f(x)$
0	-2
0.5	-6

dec.

4. $\lim_{x\to 2} \frac{x^2-4}{x-2} = \lim_{x\to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}}$

$= \lim_{x\to 2} (x+2)$

$= 2+2 = 4$

5. $\lim_{x\to 2^\pm} \frac{x+1}{x-2} =$

*odd V.A. @ $x=2$

$\lim_{x\to 2^+} \frac{x+1}{x-2} = \infty$

$\lim_{x\to 2^-} \frac{x+1}{x-2} = -\infty$

x	$f(x)$
3	4
2.5	7

6. $\lim_{x\to 3^\pm} \frac{x^2-3x-1}{x-3} =$

* odd V.A. @ $x=3$

Limits at Infinity

Find the following horizontal asymptotes:

1. $f(x) = \frac{5x^4-3x^2+2}{10x^4+3}$

H.A. @ $y = \frac{5}{10} = \frac{1}{2}$

2. $f(x) = \frac{5x^4-3x^2+2}{10x^5+3}$

H.A. @ $y=0$

3. $f(x) = \frac{5x^4-3x^2+2}{10x^3+3}$

No H.A.
(slant asymptote)

Find the following limits:

1. $\lim_{x\to\infty} \frac{5x^4-3x^2+2}{10x^4+3} = \frac{1}{2}$

consider:

$\lim_{x\to\infty} \frac{5x^4}{10x^4}$

$= \lim_{x\to\infty} \frac{1}{2}$

$= \frac{1}{2}$

2. $\lim_{x\to -\infty} \frac{5x^4-3x^2+2}{10x^5+3} = 0$

consider:

$\lim_{x\to -\infty} \frac{5x^4}{10x^5}$

$= \lim_{x\to -\infty} \frac{1}{2x}$

$= 0$

3. $\lim_{x\to\infty} \frac{5x^4-3x^2+2}{10x^3+3} = \infty$ (DNE)

consider:

$\lim_{x\to\infty} \frac{5x^4}{10x^3}$

$= \lim_{x\to\infty} \frac{x}{2}$

$= \infty$