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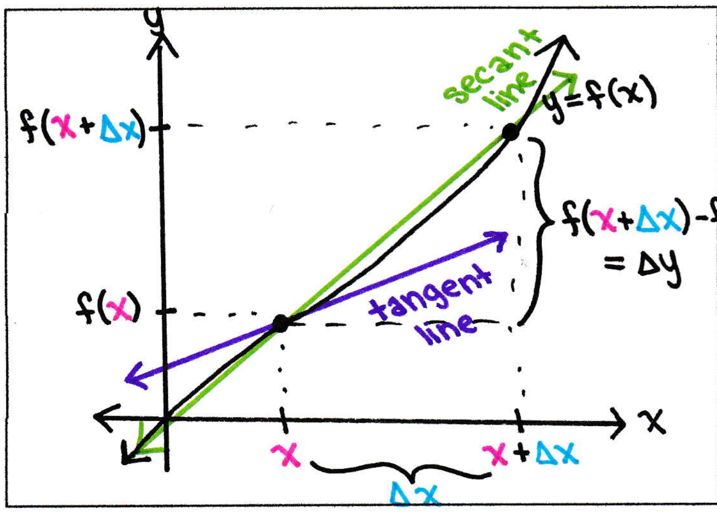
## Lesson 1.4: Limit Definition of the Derivative

Any non vertical has the same slope at every point. In calculus, we frequently deal with the slope of a curve.

The slope of a curve is defined to be the same as the slope of the curve's tangent line at a given point.

To find the slope of a tangent line, we use a limit of the slope of a secant line.




$$m_{sec} = \frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$m_{tan} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

The slope of a tangent line is called the derivative of the function at a given x-value.

The most commonly used symbol for the derivative is $f'(x)$.

**Here are some other notations you will encounter. (Assume $y = f(x)$)**

$$f'(x) = y' = \frac{dy}{dx} = \frac{d}{dx}f(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

A vertical tangent line has no slope, so a curve has no derivative at any point where it has a vertical tangent line.

Differentiation is the process of finding derivatives. If a derivative exists at a point on a curve, the function is said to be **differentiable**.

**Alternate Form of the Limit Definition of Derivative**

* gives the value of the derivative at a single point.

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

<u>Examples</u>:

1. If $f(x) = x^2 + 2$

   a. Find $f'(x)$.

$$f'(x) = \lim_{\Delta x \to 0} \frac{[(x+\Delta x)^2 + 2] - [x^2 + 2]}{\Delta x} = \lim_{\Delta x \to 0} \frac{[\cancel{x^2} + 2x\Delta x + (\Delta x)^2 + \cancel{2}] - [\cancel{x^2} + \cancel{2}]}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2x\Delta x + (\Delta x)^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} 2x - \Delta x$$

$$= 2x$$

   b. Use your answer from part a to find: $f'(-3)$.

$$f'(x) = 2x$$

$$f'(-3) = 2(-3) = -6$$

2. If $y = \sqrt{x}$, find $y'$.

$$y' = \lim_{h \to 0} \frac{[\sqrt{x+h}] - \sqrt{x}}{h} \cdot \frac{(\sqrt{x+h} + \sqrt{x})}{(\sqrt{x+h} + \sqrt{x})} = \lim_{h \to 0} \frac{(x+h) - x}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

$$= \frac{1}{2\sqrt{x}}$$

3. Given $y = f(t) = \frac{2}{t}$

Find the derivative of $y$ **with respect to $t$**.

$$\frac{dy}{dt} = f'(t) = \lim_{h \to 0} \frac{\left(\frac{2}{t+h}\right) - \left(\frac{2}{t}\right)}{h} = \lim_{h \to 0} \frac{\frac{2t}{t(t+h)} - \frac{2(t+h)}{t(t+h)}}{h}$$

$$= \lim_{h \to 0} \frac{-2\cancel{h}}{t(t+h)} \cdot \frac{1}{\cancel{h}}$$

$$= \lim_{h \to 0} \frac{-2}{t(t+h)}$$

$$= \frac{-2}{t^2}$$