Entry #: ______

Lesson 1.5: Calculating Limits Using Limit Laws & The Squeeze Theorem

Limit Laws

Sum Law	
Difference Law	
Constant Multiple Law	
Product Law	
Quotient Law	
Power Law	
Limit of a Constant	

*These limit laws are also valid for one-sided limits.

Examples: Evaluate using limit laws.

1. $\lim_{x \to 5}(2x^2 - 3x + 4)$

2. $\lim_{x \to -2} \frac{x^3 + 2x^2 - 1}{5 - 3x}$

Limit Theorems

The Squeeze Theorem:

Example:

Show that $\lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right) = 0$ using the squeeze theorem.

Practice Problems:

1. Evaluate the limit and justify each step by indicating the appropriate limit law(s).

a. $\lim_{x\to 3}(5x^3 - 3x^2 + x - 6)$	b. $\lim_{x\to -1}(x^4 - 3x)(x^2 + 5x + 3)$
c. $\lim_{u\to -2}\sqrt{u^4 + 3u + 6}$	d. $\lim_{t\to 2}\left(\frac{t^2-2}{t^3-3t+5}\right)^2$
e. $\lim_{h\to 0}\frac{(2+h)^3-8}{h}$	f. $\lim_{h\to 0}\frac{\sqrt{9+h}-3}{h}$
g. $\lim_{u\to 2}\frac{\sqrt{4u+1}-3}{u-2}$	h. $\lim_{t\to 0}\frac{\sqrt{1+t}-\sqrt{1-t}}{t}$
i. $\lim_{t\to 0}\left(\frac{1}{t\sqrt{1+t}} - \frac{1}{t}\right)$	j. $\lim_{h\to 0}\frac{4(x+h)-4x}{h}$

2. a. Use the Squeeze Theorem to show that $\lim_{x \to 0}\left(x^2 \cos 20\pi x\right) = 0$.

b. Illustrate by graphing the functions $f(x) = -x^2$, $g(x) = x^2 \cos 20\pi x$, *and* $h(x) = x^2$ on the same screen. Sketch the graph below.

3. Prove that $\lim_{x \to 0} x^4 \cos \frac{2}{x} = 0.$