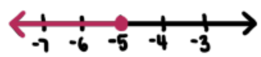
ENTRY #7

Lesson 1.7: Inequalities

WARM UP

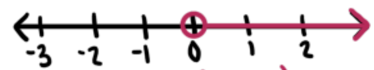
Directions: For each of the inequalities below, draw a number line that represents all possible values of x.

1. $x \leq -5$



$(-\infty, -5]$

2. $0 < x$



$(0, \infty)$

LINEAR *inequalities*

An inequality looks like an equation; however, instead of an equal sign there is one of the following symbols: $\leq$, $\geq$, $<$ and $>$.

When solving an inequality, you are finding all possible values for x that make the inequality true .

In order to solve a linear inequality, we need to isolate the variable on one side of the equation; however not all rules for solving are the same for equations and inequalities.

You can add or subtract from both sides of an inequality just like you do when solving equations.

You can also multiply both sides of an inequality by a positive quantity.

However, when multiplying by a negative quantity, you must flip the sign.

Example: $-2 < 3$ is a true statement. Multiply both sides of the inequality by -1. Is it still true? What do you need to do to make it true?

$-2 < 3$

$(-1)(-2) < (-1)(3)$

$2 \not< -3$ ← not true

$2 > -3$ is true

* must flip the inequality sign

When taking the reciprocal of positive quantities on both sides, you must also flip the sign of the inequality.

Example: $1 < 3$ is a true statement. Take the reciprocal of both sides. Is it still true? What do you need to do to make it true?

$1 < 3$

$\frac{1}{1} < \frac{1}{3}$

$1 \not< \frac{1}{3}$ ← not true

$1 > \frac{1}{3}$ is true

*must flip the sign.

SOLVING LINEAR *inequalities*

Examples: Solve the following inequalities for x. State your final answer in interval notation.

1. $4x \leq 7x - 12$

$-7x \quad -7x$

$\frac{-3x}{-3} \leq \frac{-12}{-3}$

$x \geq 4$

$[4, \infty)$

2. $-3 \leq 2x + 1 < 4$

$-1 \quad -1 \quad -1$

$\frac{-4}{2} \leq \frac{2x}{2} < \frac{3}{2}$

$-2 \leq x < \frac{3}{2}$

$[-2, \frac{3}{2})$

SOLVING NONLINEAR *inequalities*

Step 1	Move all terms to one side of the equation.
Step 2	Factor the nonzero side of the inequality.
Step 3	Determine the values that make each factor zero. (these will separate your interval)
Step 4	Make a table or diagram. Test a value on each sub interval for each factor and record the sign (+ or −)
Step 5	Determine the sign of the product of all factors on each sub-interval. State the interval that satisfies the inequality.

Examples: Solve the following inequalities for x. State your final answer in interval notation.

1. $(x+1)(x-4) \leq 0$

$x = -1$ OR $x = 4$

$x=-2$ $\quad -1 \quad$ $x=0$ $\quad 4 \quad$ $x=5$

	$x=-2$	$x=0$	$x=5$
$x+1$	−	+	+
$x-4$	−	−	+
$(x+1)(x-4)$	+	−	+

$[-1, 4]$

2. $x^2 + 5x > -6$

$+6 \quad +6$

$x^2 + 5x + 6 > 0$

$(x+3)(x+2) > 0$

$x = -3$

$x = -2$

$x=-4$ $\quad -3 \quad$ $x=-\frac{5}{2}$ $\quad -2 \quad$ $x=0$

	$x=-4$	$x=-\frac{5}{2}$	$x=0$
$x+3$	−	+	+
$x+2$	−	−	+
$(x+3)(x+2)$	+	−	+

$(-\infty, -3) \cup (-2, \infty)$

SOLVING NONLINEAR *inequalities*	See below

Examples: Solve the following inequalities for x. State your final answer in interval notation.

3. $x\boxed{(x-2)^2}(x+1) \ge 0$

Leave as one factor

$x=0 \quad x=2 \quad x=-1$

	$x=-2$	$x=-\frac{1}{2}$	$x=1$	$x=3$
		-1	0	2
x	$-$	$-$	$+$	$+$
$(x-2)^2$	$+$	$+$	$+$	$+$
$x+1$	$-$	$+$	$+$	$+$
$x(x-2)^2(x+1)$	$+$	$-$	$+$	$+$

$(-\infty,-1] \cup [0,\infty)$

4. $\frac{1+x}{1-x} > 1$

$\quad -1 \quad -1$

$\frac{1+x}{1-x} - 1 > 0$

$\frac{1+x}{1-x} - \left(\frac{1-x}{1-x}\right) > 0$

$\frac{1-1+x+x}{1-x} > 0$

$\frac{2x}{1-x} > 0$

$2x=0$ when $x=0$

$1-x=0$ when $x=1$

note $x=1$ is not in the domain

	$x=-1$	$x=\frac{1}{2}$	$x=2$
		0	1
$2x$	$-$	$+$	$+$
$1-x$	$+$	$+$	$-$
$\frac{2x}{1-x}$	$-$	$+$	$-$

$(0,1)$

ABSOLUTE VALUE *inequalities*	See below

We can rewrite absolute value inequalities to make solving them easier.

Inequality	Equivalent Form	Graph
$\lvert x\rvert < c$	$-c < x < c$	$-c \quad 0 \quad c$
$\lvert x\rvert \le c$	$-c \le x \le c$	$-c \quad 0 \quad c$
$\lvert x\rvert > c$	$x < -c$ OR $x > c$	$-c \quad 0 \quad c$
$\lvert x\rvert \ge c$	$x \le -c$ OR $x \ge c$	$-c \quad 0 \quad c$

Example: Solve each of the following inequalities for x.

1. $\lvert 2x+9\rvert \le 5$

$-5 \le 2x+9 \le 5$

$-9 \qquad -9 \qquad -9$

$\frac{-14}{2} \le \frac{2x}{2} \le \frac{-4}{2}$

$-7 \le x \le -2$

$[-7,-2]$

2. $\lvert 3x+12\rvert > 3$

$3x+12 > 3$ OR $3x+12 < -3$

$-12 \quad -12 \qquad -12 \quad -12$

$\frac{3x}{3} > \frac{-9}{3} \qquad \frac{3x}{3} < \frac{-15}{3}$

$x > -3$ OR $x < -5$

$(-\infty,-5) \cup (-3,\infty)$