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Lesson 10.10: Interval of Convergence (by Ratio Test)

The radius of convergence of a power series is the distance from the center c at which the series will converge.

The **interval of convergence** is the range of x-values within which the series will converge.

Examples: Find the interval of convergence and the radius of convergence of the following power series.

1. $\sum_{n=1}^{\infty} \frac{2^n x^n}{n}$

$$\lim_{n\to\infty}\left|\frac{2^{n+1}x^{n+1}}{n+1}\cdot\frac{n}{2^n x^n}\right| = \lim_{n\to\infty}\left|\frac{2\cdot\cancel{2^n}\cdot x\cdot\cancel{x^n}}{n+1}\cdot\frac{n}{\cancel{2^n}\cancel{x^n}}\right| = \lim_{n\to\infty}\left|\frac{2xn}{n+1}\right| = |2x|$$

$$|2x|<1 \Rightarrow \frac{-1}{2}<\frac{2x}{2}<\frac{1}{2}$$

$$-\tfrac{1}{2}<x<\tfrac{1}{2} \quad \text{*✓ endpoints...}$$

Check:

$x=\frac{1}{2}$

$$\sum_{n=1}^{\infty}\frac{2^n\left(\frac{1}{2}\right)^n}{n} = \sum_{n=1}^{\infty}\frac{1}{n}$$ Harmonic series diverges

$x=-\frac{1}{2}$

$$\sum_{n=1}^{\infty}\frac{2^n\left(-\frac{1}{2}\right)^n}{n} = \sum_{n=1}^{\infty}\frac{(-1)^n}{n}$$ Alternating Harmonic series converges

$\Rightarrow$ I.O.C: $[-\frac{1}{2}, \frac{1}{2})$

R.O.C: $\frac{1}{2}$

2. $\sum_{n=0}^{\infty} \frac{(x+1)^n}{2^n} = \sum_{n=0}^{\infty}\left(\frac{x+1}{2}\right)^n$ ← Geometric series (don't have to ✓ endpoints)

$$\left|\frac{x+1}{2}\right|<1$$

$$\Rightarrow -1<\frac{x+1}{2}<1$$

$$-2<x+1<2$$

$$-3<x<1$$

I.O.C: $(-3, 1)$

R.O.C: 2

3. $\sum_{n=0}^{\infty} n!\,x^n$

Converges only when $x=0$.

4. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$

$$\lim_{n\to\infty}\left|\frac{(-1)^{n+1}x^{2(n+1)}}{(2(n+1))!}\cdot\frac{(2n)!}{(-1)^n x^{2n}}\right|$$

$$= \lim_{n\to\infty}\left|\frac{(-1)^n(-1)x^{2n+2}}{(2n+2)!}\cdot\frac{(2n)!}{(-1)^n x^{2n}}\right|$$

$$= \lim_{n\to\infty}\left|\frac{\cancel{(-1)^n}(-1)\cancel{x^{2n}}x^2}{(2n+2)(2n+1)\cancel{(2n)!}}\cdot\frac{\cancel{(2n)!}}{\cancel{(-1)^n}\cancel{x^{2n}}}\right|$$

$$= \lim_{n\to\infty}\left|\frac{x^2}{(2n+2)(2n+1)}\right| = 0 < 1$$

$$\Rightarrow \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n)!} \text{ always converges.}$$

Note: In previous lessons, we found intervals of convergence for Geometric Series & it was not necessary to check endpoints because Geometric Series diverge when $r=1$. However, the ratio test is inconclusive when $r=1$, so you must check the endpoints.