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Lesson 10.11: Absolute vs Conditional Convergence

Many convergent series have negative terms, alternating or some pattern. Taking the absolute value of each term of a convergent series with some negative terms makes a new positive series less likely to converge since the sum will be greater without negative terms.

If the new positive series is still convergent the original series is called absolutely convergent.

If the new positive series diverges the original series with negative terms is called conditionally convergent.

If a series converges after taking the absolute value of its terms it is guaranteed to also converge with no absolute value.

Examples:

1. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ ← alternating harmonic series converges

$\sum_{n=1}^{\infty} \frac{1}{n}$ ← harmonic series diverges

$\Rightarrow \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ is conditionally convergent.

2. $\sum_{n=1}^{\infty} \frac{\cos(n)}{n^2} = \sum_{n=1}^{\infty} \cos(n) \frac{1}{n^2}$ $\quad p = 2 > 1$

$\Rightarrow \sum_{n=1}^{\infty} \frac{\cos(n)}{n^2}$ converges by PST

$\sum_{n=1}^{\infty} \frac{|\cos(n)|}{n^2}$ compare to: $\sum_{n=1}^{\infty} \frac{1}{n^2}$ $\quad p = 2 > 1 \Rightarrow \sum_{n=1}^{\infty} \frac{1}{n^2}$ converges by PST.

$\frac{|\cos(n)|}{n^2} < \frac{1}{n^2} \Rightarrow \sum_{n=1}^{\infty} \frac{|\cos(n)|}{n^2}$ converges by DCT

$\Rightarrow \sum_{n=1}^{\infty} \frac{\cos(n)}{n^2}$ is absolutely convergent.

3. $\sum_{n=1}^{\infty} \frac{(-e)^n}{n^e}$

$\lim_{n\to\infty} \left| \frac{(-e)^{n+1}}{(n+1)^e} \cdot \frac{n^e}{(-e)^n} \right|$

$= \lim_{n\to\infty} \left| \frac{\cancel{-e^n}(-e)}{(n+1)^e} \cdot \frac{n^e}{\cancel{(-e)^n}} \right|$

$= \lim_{n\to\infty} \left| \frac{-e\, n^e}{(n+1)^e} \right| = |-e| = e > 1$

$\Rightarrow \sum_{n=1}^{\infty} \frac{(-e)^n}{n^e}$ diverges by RT