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Lesson 10.3: Power Series, Geometric Power Series, & Integration and Differentiation of Power Series

Power Series

A power series is a series with _________________ terms like:

The series above is a ___________________ power series. In order for the series above to converge, what must be true about x?

This means, for these x −values, the function $f(x) = \frac{1}{1-x}$ can be written as:

Examples: Find a power series for each of the following functions. Show four terms and the general term. Also give the series using sigma notation and give the interval of convergence.

1. $f(x) = \frac{1}{1+x}$

2. $g(x) = \frac{3}{1-2x}$

3. $h(x) = \frac{1}{3x}$

Power Series by Substitution

Examples:

1. Using the power series from Example 1 on the previous page, make a new power series for $f(x^2)$.

2. Using the power series from Example 2 on the previous page, make a new series for $g(\sqrt{x})$.

Power Series by Differentiation

Examples:

1. Use the power series for $f(x) = \frac{1}{1-x}$ on the previous page to write a power series for $f'(x)$.

2. If $j(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots + \frac{x^n}{n!} + \cdots$ find $j'(x)$.

   Can you identify the function $j(x)$?

Power Series by Integration

Examples:

1. Since $\frac{1}{1+t} = 1 - t + t^2 - t^3 + \cdots + (-1)^n t^n + \cdots$

   $\int_0^x \frac{1}{1+t}\, dt = \int_0^x (1 - t + t^2 - t^3 + \cdots + (-1)^n t^n + \cdots)\, dt$ Now, integrate both sides.

2. Use the result of the example above to write a power series for $f(x) = \ln(x)$.

3. Now, write a series for $g(x) = x^2 \ln(x)$.