Entry #: _____

Lesson 10.4: Taylor Series

In this lesson, we will be finding polynomial functions that can be used to approximate transcendental functions

If $P(x)$ is a ___________________ function used to ________________________ some other function $f(x)$, they must contain the same point with some x −value c.

That means that $P(c) = f(c)$ for some value of c.

To be a better approximation they should have the same slope at that point.

That means that $P'(c) = f'(c)$ for some value of c.

For even greater accuracy, $P''(c) = f''(c)$ and so on…..

Putting all of this together, gives us the **Taylor Polynomial Expansion**:

When the center is at $c = 0$, the Taylor polynomial is called a **Maclaurin Polynomial** which can be written as:

Example:

1. Use the definition of a Maclaurin Polynomial to find the fifth degree Maclaurin Polynomial for $f(x) = e^x$.

$f(x) =$	$f(0) =$
$f'(x) =$	$f'(0) =$
$f''(x) =$	$f''(0) =$
$f'''(x) =$	$f'''(0) =$
$f^{(4)}(x) =$	$f^{(4)}(0) =$
$f^{(5)}(x) =$	$f^{(5)}(0) =$

$P(x) =$

This polynomial is a good approximation for $f(x) = e^x$. By extending the pattern into an infinite series, it becomes exactly correct instead of an approximation.

$f(x) = e^x =$

The general form of Taylor and Maclaurin Polynomials an be extended to Taylor and Maclaurin Series.

Taylor Series (Provided $f(x)$ has derivatives of all orders.)

Maclaurin Series

These formulas allow us to form a power series for functions that cannot be written as geometric power series.

Examples:

1. Use the definition of a Taylor Polynomial to find the fourth order Taylor Polynomial and the Taylor Series for $f(x) = \cos(x)$ centered at $c = \pi$.

$f(x) =$	$f(\pi) =$
$f'(x) =$	$f'(\pi) =$
$f''(x) =$	$f''(\pi) =$
$f'''(x) =$	$f'''(\pi) =$
$f^{(4)}(x) =$	$f^{(4)}(\pi) =$

$P_4(x) =$

$f(x) =$

2. Use your Taylor Polynomial from the example above to approximate $\cos(3)$.

3. Use the definition of a Maclaurin Series to find the Maclaurin series for $f(x) = \sin(x)$.

$f(x) =$	$f(0) =$
$f'(x) =$	$f'(0) =$
$f''(x) =$	$f''(0) =$
$f'''(x) =$	$f'''(0) =$
$f^{(4)}(x) =$	$f^{(4)}(0) =$
$f^{(5)}(x) =$	$f^{(5)}(0) =$

$f(x) = \sin(x) =$

4. Find a power series for $f(x)$ centered at $c = 1$ if $f(1) = 2$ and $f^{(n)}(1) = n!$.