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Lesson 10.6: Error Approximations

Alternating Series Remainder

For a convergent alternating series when ______________________ the sum of a series by using only the first *n* terms, the error will be less than or equal to the absolute value of the $(n+1)^{st}$ term (this is the next term or the first unused term).

Examples:

1. Approximate the sum of $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n!}$ by using the first 6 terms.

2. Find the upper bound for the remainder for the approximation from Example 1.

3. Find the upper and lower bounds for the actual sum of the series in Example 1.

4. Approximate $\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}$ with an error of less than .001.

5. Use an elementary series to find the actual value of the series in Example 4.

The **Alternating Series Remainder** can be written as:

Lagrange Remainder

If a non-alternating series is approximated, the method is slightly different and slightly more difficult.

This remainder is called the Lagrange Remainder or ________________________________ .

As in an alternating series remainder the $(n+1)^{st}$ term of the Taylor Series is used; however, the $(n+1)^{st}$ derivative factor is carefully chosen.

Choose the value of z which makes the $\left|f^{(n+1)}(z)\right|$ factor a maximum. This may be at the center, at the x-value where *f* is to be evaluated, or you may know the maximum value in advance (sine and cosine functions have a maximum value of 1).

Examples:

1. Estimate e^2 using a Maclaurin polynomial of degree 10 for e^x.

2. Use the Lagrange form of the remainder (error) to estimate the accuracy of using the partial sum in the previous example.

3. If $f^{(5)}(x) = 700\sin(x)$ and if $x = .7$ is in the convergence interval for the power series of f centered at $x = 0$, find an upper limit for the error when the fourth degree Taylor polynomial is used to approximate $f(.7)$.

4. If $f^{(6)}(x)$ is a positive decreasing function, find the error bound when a 5th degree Taylor polynomial centered at $x = 4$ is used to approximate $f(4.1)$. Assume the series converges for $x = 4.1$.