Entry #: _____

Lesson 2.1: Differentiation Rules, Tangent Lines, Differentiability, & Rates of Change

Derivative Rules

Power Rule	
Constant Rule	
Scalar Multiple Rule	
Sum Rule	

Examples: Differentiate.

1. $f(x) = x^4$
   $f'(x) =$
2. $y = x^{-\frac{2}{3}} + 3$
   $y' =$
3. $h(t) = 5 - \frac{1}{2t^3}$
4. $f(x) = \frac{5}{(2x)^3}$

Higher-Order Derivatives

Since the derivative of a function is another _____________ , we can repeat the differentiation process to find the derivative of a _______________ . The result is still another function which could again be differentiated. These derivatives are called ________________ derivatives.

Notation:

First Derivative				
Second Derivative				
Third Derivative				
Fourth Derivative				

Example: For $f(x) = \frac{1}{2\sqrt[3]{x^2}}$, find $f'(1)$ and $f''(8)$.

Equation of a Tangent Line

Since the derivative of a function gives us a __________ formula for tangent lines to the graph of the function, the derivative can be used to find equations of tangent lines.

Sometimes, we will want to find a line perpendicular to the tangent line at a certain point. Such a line is called a __________________ .

$$m_{normal} = \frac{-1}{m_{tangent}}$$

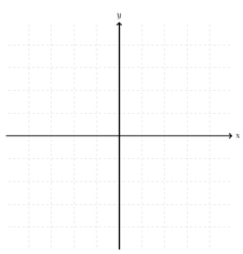
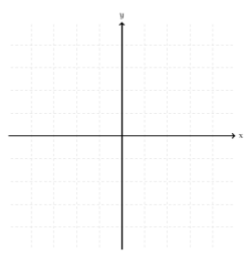
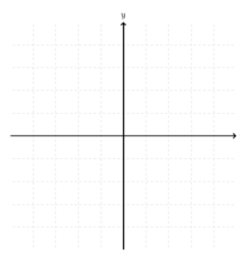
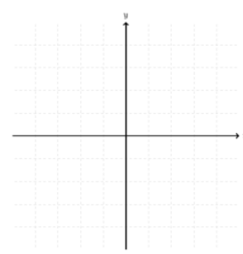
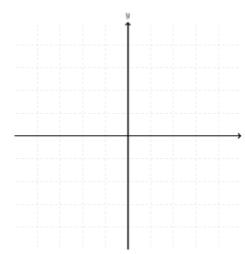
Examples:

1. Find an equation of the line tangent to the graph of $f(x) = 4x^5 - 3x^2 + 5$ at the point $(1,6)$.

2. Find an equation of the normal line to the same curve at the same point.

Non-differentiability (When a derivative does not exist)

Each of these functions has no derivative when $x = 1$.

These five characteristics destroy differentiability:

1. Holes
2. Jumps (breaks)
3. Vertical Asymptotes
4. Sharp Turns
5. Vertical Tangent Lines

Note: If a function is not continuous, it is not differentiable (see the first three figures above). A function may be continuous and still not be differentiable (see the last two figures above).

Examples: Find the x-values where $f(x)$ is <u>not</u> differentiable. Give a reason for each.

1. $f(x) = |x|$
2. $f(x) = \begin{cases} x^2, & x \le 0 \\ x, & x > 0 \end{cases}$
3. $f(x) = \begin{cases} x^2, & x \le 0 \\ x^2 + 1, & x > 0 \end{cases}$

Rate of Change

Another meaning of slope is _________________ . We now have two ways to find slopes (rates of change).

1. **Average Rate of Change:** This is the slope between two points. It is found without using a derivative (algebraically).

   AROC =

2. **Instantaneous Rate of Change:** This is the slope at a single point. It is usually found by using a derivative (calculus).

   IROC =

Examples:

1. If $f(x) = x^3 + 2x$, find the average rate of change from $x = 10$ to $x = 30$.

2. If $f(x) = x^3 + 2x$, find the instantaneous rate of change when $x = 10$.