LESSON 2.1: WHAT IS A FUNCTION?

WARM UP

Let $f(x) = -2x^2 + 3$. Calculate each of the following:

a. $f(0) = -2(0)^2 + 3$
$= -2(0) + 3$
$= \boxed{3}$

b. $f(-1) = -2(-1)^2 + 3$
$= -2(1) + 3$
$= -2 + 3$
$= \boxed{1}$

c. $f\left(\frac{1}{4}\right) = -2\left(\frac{1}{4}\right)^2 + 3$
$= -2\left(\frac{1}{16}\right) + 3$
$= -\frac{1}{8} + \frac{24}{8}$
$= \boxed{-\frac{23}{8}}$

d. $f\left(-\frac{3}{2}\right) = -2\left(-\frac{3}{2}\right)^2 + 3$
$= -2\left(\frac{9}{4}\right) + 3$
$= -\frac{9}{2} + 3$
$= -\frac{9}{2} + \frac{6}{2}$
$= \boxed{-\frac{3}{2}}$

DEFINITION OF A *function*

A __function__ is a relation that assigns each input with exactly one __output__.

Example: Function

x	0	2	5	-1	-2
y	1	3	1	2	-5

Example: Not a Function *same input → two different outputs

x	0	2	6	0	-2
y	1	5	-4	3	5

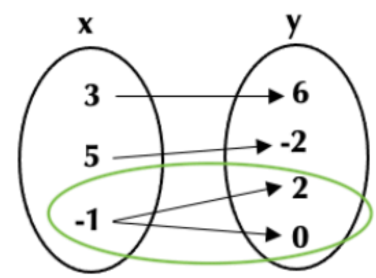
Examples: Determine if each of the relations below are functions or not.

1.

x	-3	-2	0	1	5	-3
y	2	5	3	-1	5	2

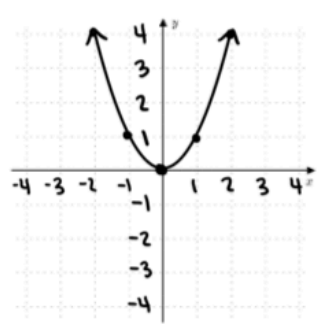
Each input has exactly one output, so this is a function.

2.

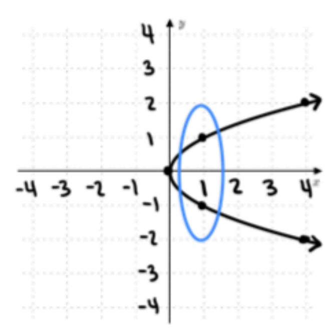


Not a function because $x=-1$ has two different outputs.

3. $y = x^2$ Function



4. $x = y^2$ Not a function.



*all inputs (except $x=0$) has two outputs.

EVALUATING *a function*

We often write equations in function notation so that we can clearly distinguish the input from the output.

For example, we can write $y = -2x + 1$ as $f(x) = -2x + 1$.

Where x is the input and $f(x)$ is the output.

Examples: Let $f(x) = x^2 + 1$. Evaluate each of the following.

1. $f(-3) = (-3)^2 + 1$
   $= 9 + 1$
   $= \boxed{10}$

2. $f(-a) = (-a)^2 + 1$
   $= \boxed{a^2 + 1}$

3. $f(2c + 1) = (2c+1)^2 + 1$
   $= (4c^2 + 4c + 1) + 1$
   $= \boxed{4c^2 + 4c + 2}$

4. $\frac{f(a+h) - f(a)}{h}, h \neq 0$
   $= \frac{(a+h)^2 + 1 - [a^2 + 1]}{h}$
   $= \frac{\cancel{a^2} + 2ah + h^2 + \cancel{1} - \cancel{a^2} - \cancel{1}}{h}$
   $= \frac{2ah + h^2}{h}$
   $= \boxed{2a + h}$

WAYS TO REPRESENT *a function*

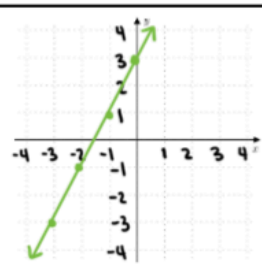
Verbal (Words)

The sum of 3 and twice a number.

Algebraic (Equation)

$y = 2x + 3$

Visual (Graph)



Numerical (Table)

x	-2	-1	0	1	2	3
y	-1	1	3	5	7	9

DOMAIN OF A *function*

Recall, the domain of a function is the set of all possible x-values (inputs).

We can find the domain of a function by determining which x values would result in an undefined or imaginary output.

For example, in the equation $y = \sqrt{x}$, any negative real number would result in an imaginary output. Therefore, the domain would be $[0, \infty)$.

Examples: Find the domain (in interval notation) of each of the functions below.

1. $f(x) = \frac{1}{x^2 - x} = \frac{1}{x(x-1)}$ can't divide by zero

   $x(x-1) \neq 0$

   $x \neq 0$ OR $x - 1 \neq 0$

   $+1 \quad +1$

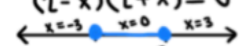
   $x \neq 1$

   D: $(-\infty, 0) \cup (0, 1) \cup (1, \infty)$

2. $g(x) = \sqrt{4 - x^2}$

   $4 - x^2$ can't be < 0

   $(2-x)(2+x) \geq 0$

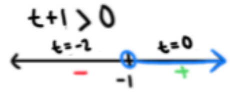
   

$2-x$	+	+	−
$2+x$	−	+	+
$(2-x)(2+x)$	−	+	−

   D: $[-2, 2]$

3. $h(t) = \frac{t}{\sqrt{t+1}}$

   $t + 1 > 0$

   D: $(-1, \infty)$

4. $f(x) = \frac{\sqrt{x-1}}{x^2 - 9} = \frac{\sqrt{x-1}}{(x+3)(x-3)}$ can't divide by zero

   $(x+3)(x-3) \neq 0$

   $x + 3 \neq 0 \quad x - 3 \neq 0$

   $x \neq -3 \quad x \neq 3$

   $x - 1 \geq 0$

   $+1 \quad +1$

   $x \geq 1$

   D: $[1, 3) \cup (3, \infty)$

PIECEWISE *functions*

A piecewise function is a function comprised of different functions put together on particular x-values.

Example: Evaluate the following if $f(x) = \begin{cases} -x - 5 & x < -2 \\ x^2 & -2 \leq x < 2 \\ 2x & x \geq 2 \end{cases}$

1. $f(-4) = -(-4) - 5 = 4 - 5 = \boxed{-1}$

2. $f(0) = (0)^2 = \boxed{0}$

3. $f(2) = 2(2) = \boxed{4}$

4. $f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 = \boxed{\frac{1}{4}}$