Entry #: 8

Lesson 2.2: Product Rule, Quotient Rule, & Trig Rules

Product Rule & Quotient Rule

Product Rule	$\frac{d}{dx}(f(x) \cdot g(x)) = f(x)g'(x) + f'(x)g(x)$	$\frac{d}{dx}(f \cdot g) = fg' + f'g$
Quotient Rule	$\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$	$\frac{d}{dx}\frac{f}{g} = \frac{gf' - fg'}{g^2}$

Examples: Differentiate

1. $f(x) = (3x^2 - 2)(2x + 3)$

$f'(x) = (3x^2 - 2)(2) + (6x)(2x+3)$

$= 6x^2 - 4 + 12x^2 + 18x$

$= 18x^2 + 18x - 4$

2. $y = \frac{2x^2 - 4x + 3}{2 - 3x} = \frac{(2-3x)(4x-4) - (2x^2 - 4x + 3)(-3)}{(2-3x)^2}$

3. $y = \frac{-9}{5x^2} = \frac{(5x^2)(0) - (-9)(10x)}{(5x^2)^2}$

$= \frac{90x}{25x^4} = \frac{18}{5x^3}$

4. If $f(2) = 3$ and $f'(2) = -4$
Find $g'(2)$ when $g(x) = x^2 f(x)$.

$g'(x) = (2x)(f(x)) + (x^2)(f'(x))$

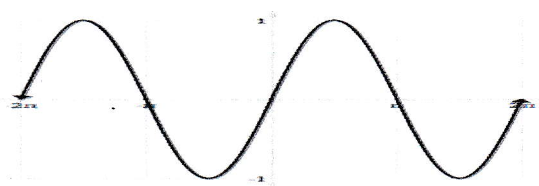
$g'(2) = 2(2)(f(2)) + (2^2)(f'(2))$

$= 4(3) + 4(-4)$

$= 12 + (-16) = -4$

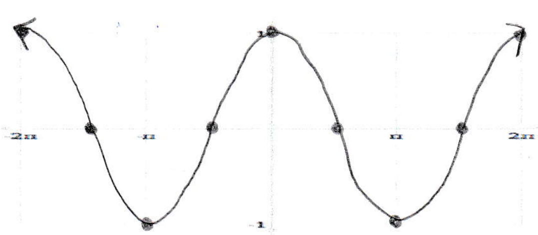
Trig Rules

The graph of $f(x) = \sin(x)$ is shown at right.



Estimate slopes for the graph of $f(x) = \sin(x)$ at $x = -2\pi, \frac{-3\pi}{2}, -\pi, \frac{-\pi}{2}, 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \& \ 2\pi$.

Plot these slopes in the coordinate plane at right, and connect them to make a smooth, continuous curve.
This is the graph of $f'(x)$.



$f'(x) = \cos(x)$

The only derivatives you need to memorize are sin(x) & cos(x)

Derivatives of Trigonometric Functions

$\frac{d}{dx}\sin(x) = \cos(x)$
$\frac{d}{dx}\cos(x) = -\sin(x)$
$\frac{d}{dx}\tan(x) = \sec^2(x)$
$\frac{d}{dx}\cot(x) = -\csc^2(x)$
$\frac{d}{dx}\sec(x) = \sec(x)\tan(x)$
$\frac{d}{dx}\csc(x) = -\csc(x)\cot(x)$

Examples: Differentiate each of the following.

1. $y = \frac{\sin(x)}{\cos(x)}$ $= \frac{(\cos(x))(\cos(x)) - (\sin(x))(-\sin(x))}{(\cos(x))^2} = \frac{\cos^2(x) + \sin^2(x)}{(\cos^2(x))}$

$$= \frac{1}{\cos^2(x)} = \sec^2(x)$$

2. $f(x) = 2x\cos(x)$ $= (2x)(-\sin(x)) + 2(\cos(x))$

$$= -2x\sin(x) + 2\cos(x)$$

$$= -2(x\sin(x) - \cos(x))$$

3. Find an equation of a line tangent to the graph of $y = \frac{\sin(\theta)}{\theta}$ when $\theta = \pi$. $(\pi, 0)$

$$\frac{dy}{d\theta} = \frac{(\theta)(\cos(\theta)) - (\sin(\theta))(1)}{\theta^2}$$

$$= \frac{\theta\cos(\theta) - \sin(\theta)}{\theta^2}$$

$$m = \frac{\pi\cos(\pi) - \sin(\pi)}{(\pi)^2}$$

$$m = \frac{\pi(-1)}{\pi^2} = -\frac{1}{\pi}$$

$y = mx + b$

$0 = -\frac{1}{\pi}(\pi) + b$

$0 = -1 + b$

$b = 1$

$y = -\frac{1}{\pi}x + 1$