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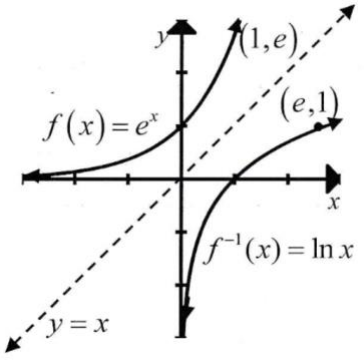
Lesson 2.3: Logarithmic Functions & Log Derivatives



The inverse of $f(x) = e^x$ is $f^{-1}(x) = \ln(x)$.

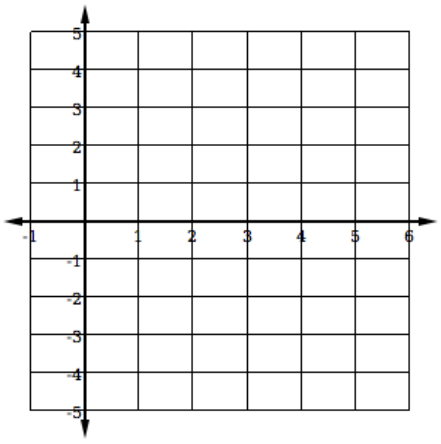
Note: $\log_e(x)$ is usually written $\ln(x)$ and $\log_{10}(x)$ is usually written simply as $\log(x)$.

Graphs of Logarithmic Functions:

If $f(x) = \log_a x$ and $a > 1$, then	
1.	2.
3.	4.

Example:

1. Without using a calculator, sketch a graph of $y = |\ln(x-2)|$. Write an equation for the graph's asymptote.



Change of Form Definition

Examples:

1. Change the following equations from exponential form to log form or vice versa.

   a. $3^4 = 81$ b. $\log(.1) = -1$

2. a. Since $e^0 = 1$, $\ln(1) =$ b. Since $e^1 = e$, $\ln(e) =$ c. $\ln(e^n) = e^{\ln(n)} =$

3. Simplify.

   a. $\ln\left(e^{\sqrt{2}}\right) =$ b. $e^{\ln(3x)} =$

   b. c. $10^{\log(2)} =$ d. $\log_2 2^{x^2} =$

Properties of Logarithms

1.
2.
3.

These properties work for any bases, but only if $a > 0$ and $b > 0$.

Examples:

1. Find using the properties of logarithms.

   a. $\ln\frac{5}{8} =$ b. $\ln\sqrt[3]{x^2+1} =$

2. Condense into a single logarithm. ($x > 0$ and $y > 0$)

   a. $-3\ln(x) + 5\ln(y)$

   b. $\frac{1}{2}\ln(x) + \ln(x+1) - 3\ln(y)$

3. Solve for x.

   a. $y = e^{2x-5} + 6$

   b. $\log_2(x) - \log_2(x-8) = 3$

Change of Base Formula

Examples:

1. Use your calculator to find $\log_7 112$ to 3 decimal places.

2. a. Find an exact value for x, if $3^{x+2} = 6$.

   b. Use your calculator to find a decimal value for your answer from Part a.

Differentiating Logarithmic Functions

Examples: Differentiate.

1. $y = \ln(5x)$
2. $f(t) = \ln(3t^2 - t)$
3. $h(x) = x \ln(x)$
4. Find $\frac{d}{dy} \ln|5 - 2y^3|$
5. Differentiate $y = \ln \frac{x\sqrt{2x+1}}{x^2+1}$
6. If $y = \log_2(x^2 + 1)$, find $y'(2)$.

Procedure for Logarithmic Differentiation

We use logarithmic differentiation when we are trying to take a derivative of a function that has a variable both in the exponent and the base.

1.
2.
3.
4.
5.

Example:

1. Differentiate $y = (x^2 + 1)^{x-1}$. Express your answer in terms of x.