ENTRY #14

Lesson 2.4: Average Rate of Change of a Function

AVERAGE RATE OF change

We calculate rate of change frequently in everyday life.

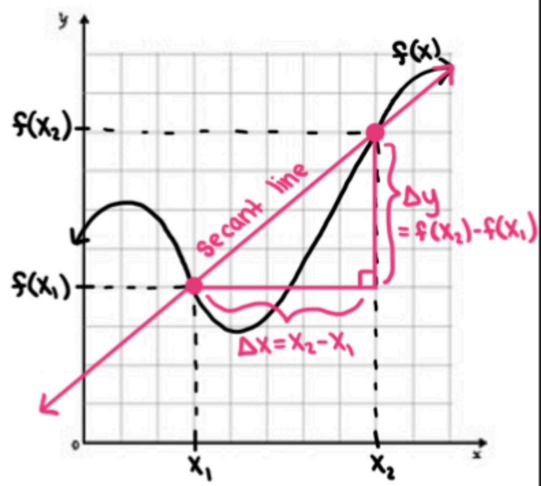
The most common example is when you are trying to figure out how long it will take you to arrive to a destination based on the distance and the speed limit.

For example, if the speed limit is 60 miles per hour and the destination is 30 miles away, how long would you estimate it would take to get to your destination?

About 30 minutes.

We could use the speed limit as an "average speed" in this scenario to simplify the problem and get a good estimate.

We could also calculate the average rate of change of a function using the slope of a secant line line.



$$\text{Average Rate of Change} = \text{Slope of Secant line} = \frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Examples:

1. For the function $f(x) = x^2 - 2x + 1$, calculate the average rate of change between the following points:

a. $x = -1$ and $x = 3$

$$\text{AROC} = \frac{f(3) - f(-1)}{3-(-1)} = \frac{4-4}{4} = \frac{0}{4} = 0$$

b. $x = -3$ and $x = 0$

$$\text{AROC} = \frac{f(0) - f(-3)}{0-(-3)} = \frac{1-16}{3} = \frac{17}{3}$$

AVERAGE RATE OF *change*

Examples:

2. Calculate the average rate of change of $f(x) = \frac{2}{x+1}$ on [1,4].

$$\text{AROC} = \frac{f(4)-f(1)}{4-1} = \frac{\frac{2}{5}-1}{3} = \frac{\frac{2}{5}-\frac{5}{5}}{3}$$
$$= \frac{-3/5}{3}$$
$$= -3/5\left(\tfrac{1}{3}\right)$$
$$= -1/5$$

3. Calculate the average rate of change of $f(x) = x^2 + 3$ between $x = a$ and $x = a + h$.

$$\text{AROC} = \frac{f(a+h)-f(a)}{(a+h)-a} = \frac{(a+h)^2+3-(a^2+3)}{h}$$
$$= \frac{\cancel{a^2}+2ah+h^2+\cancel{3}-\cancel{a^2}-\cancel{3}}{h}$$
$$= \frac{2ah+h^2}{h}$$
$$= 2a+h$$

4. Let $f(x) = 3x - 5$. Find the average rate of change of f between the following points:

a. $x = 0$ and $x = 1$

$$\text{AROC} = \frac{f(1)-f(0)}{1-0} = \frac{-2-(-5)}{1} = 3$$

b. $x = 3$ and $x = 7$

$$\text{AROC} = \frac{f(7)-f(3)}{7-3} = \frac{16-4}{4} = \frac{12}{4} = 3$$

c. $x = a$ and $x = a + h$

$$\text{AROC} = \frac{f(a+h)-f(a)}{(a+h)-a} = \frac{3(a+h)-5-(3a-5)}{h} = \frac{\cancel{3a}+3h-\cancel{5}-\cancel{3a}+\cancel{5}}{h}$$
$$= \frac{3h}{h}$$
$$= 3$$

d. What conclusion can you draw from the calculations above?

The rate of change of a linear function is constant.