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Lesson 2.4: Chain Rule

Chain Rule

If $y = f(g(x))$, then $y' =$ $f'(g(x)) \cdot g'(x)$

or in another form: $\frac{d}{dx} f(u) =$ $f'(u) \cdot u'$ (where u is a function of x)

Examples: Differentiate the following.

1. $y = (x^2 - 3)^2$

$y' = 2(x^2-3) \cdot (2x)$

$y' = 4x(x^2-3)$

$y' = 4x^3 - 12x$

2. $y = \sin(3x)$

$y' = (\cos(3x)) \cdot (3)$

$y' = 3\cos(3x)$

3. $f(x) = \sqrt{(3x^2 - x)^3} = (3x^2 - x)^{3/2}$

$f'(x) = \frac{3}{2}(3x^2 - x)^{1/2} \cdot (6x - 1)$

4. $y = \cos^2 x = (\cos(x))^2$

$y' = 2(\cos(x)) \cdot (-\sin(x))$

$y' = -2\sin(x)\cos(x)$

$y' = -\sin(2x)$

5. $g(x) = \frac{1}{2x+1} = (2x+1)^{-1}$

$g'(x) = -(2x+1)^{-2} \cdot (2)$

$g'(x) = -2(2x+1)^{-2}$

$g'(x) = \frac{-2}{(2x+1)^2}$

6. $f(t) = \sin^3(4t^2) = (\sin(4t^2))^3$

$f'(t) = 3(\sin(4t^2))^2 \cdot (-\cos(4t^2)) \cdot (8t)$

7. $y = x^2\sqrt{1 - x^2} = x^2(1-x^2)^{1/2}$

$y' = (2x)(1-x^2)^{1/2} + (x^2)\left(\frac{1}{2}(1-x^2)^{-1/2}\right)(-2x)$

8. *If* $p(2) = 5, q(2) = 3, q'(2) = 2,\ p'(3) = 4, and\ q'(3) = \frac{3}{2}, find\ \frac{d}{dx}p(q(x)) at\ x = 2.$

$$\frac{d}{dx} p(q(x))\Big|_{x=2} = p'(q(x)) \cdot q'(x)\Big|_{x=2}$$

$$= p'(q(2)) \cdot q'(2)$$

$$= p'(3) \cdot 2$$

$$= 4 \cdot 2$$

$$= 8$$