LESSON 2.4: LIMITS & CONTINUITY

CONTINUITY AT A *point*

A function is continuous if there are no breaks in the graph.

Informally, you can trace a graph without having to lift your pencil.

A break in the graph is called a discontinuity.

A function f is continuous at $x = c$ if:

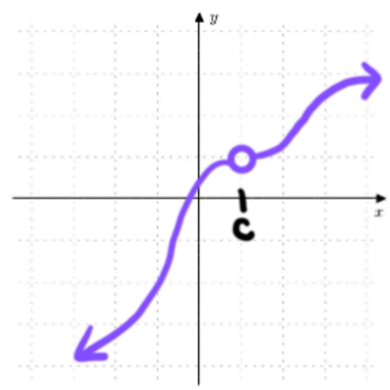
$$\lim_{x \to c} f(x) = f(c)$$

REMOVABLE *discontinuities*

If $\lim_{x \to c} f(x)$ exists but is not equal to $f(c)$, we say that f has a removable discontinuity at $x = c$.

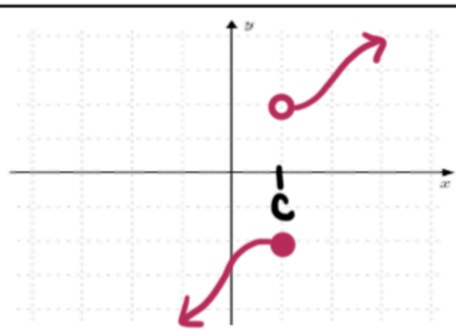
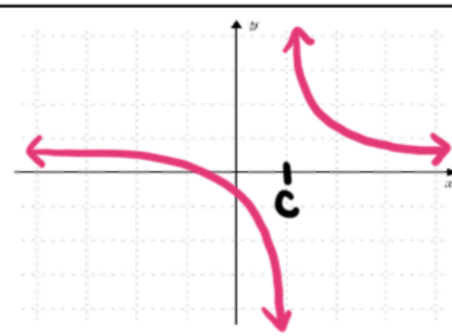
A hole is a removable discontinuity.

We call these types of discontinuities "removable" because you could easily "fill the hole" by defining $f(c)$.



NON REMOVABLE *discontinuities*

The two types of non-removable discontinuities we will study in this course are:

Jump Discontinuities	Vertical Asymptotes
Occur if the limits $\lim_{x \to c^+} f(x)$ and $\lim_{x \to c^-} f(x)$ exist but are not equal.	Occur when one or both sided limits are infinite.
	

IDENTIFYING *discontinuities*

Examples: Identify any discontinuities for the functions below.

1. $f(x) = \frac{x^2-1}{(x+5)(x-1)}$

   $= \frac{(x-1)(x+1)}{(x+5)(x-1)}$

   hole @ x=1 (removable)

   VA @ x=-5 (non-removable)

2. $f(x) = \begin{cases} \frac{1}{2}x + 4 & x < 0 \\ -1 & 0 \le x < 4 \\ x^2 - 8x + 15 & x \ge 4 \end{cases}$

   jump discontinuity at x=0 (non removable)

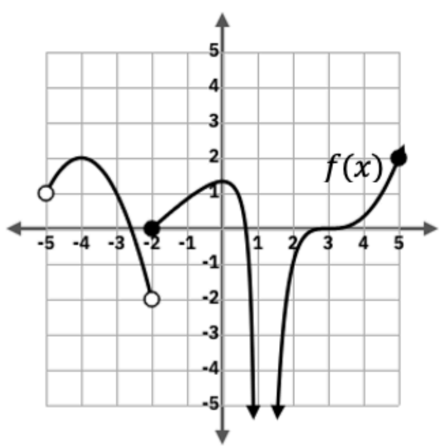
3. $f(x) = 2x^4 + x - 5$

   No discontinuities.

   * Polynomial functions are continuous.

4. $f(x) = \begin{cases} \frac{1}{x} & x < 0 \\ x^2 + 3 & x \ge 0 \end{cases}$

   infinite discontinuity at x=0 (non removable)

5. 

   hole @ x=-5 (removable)

   jump disc. @ x=-2 (non removable)

   vertical asymptote @ x=1 (non removable)

BASIC LAWS OF *continuity*

If $f(x)$ and $g(x)$ are continuous at $x = c$, then the following functions are continuous at $x = c$:

i. f(x) + g(x) & f(x)-g(x)

ii. f(x) · g(x)

iii. kf(x) for any constant k

iv. f(x)/g(x) if g(c) ≠ 0

Also, if g is continuous at $x = c$, and f is continuous at $x = g(c)$, then the composite function __f(g(x))__ is continuous at __x=c__ .

USING CONTINUITY TO EVALUATE *limits*

If a function is ___continuous___ at $x = c$, then: $\lim_{x \to c} f(x) = f(c)$

However, if a function is not continuous at $x = c$ direct substitution will not work.

Examples: Use the continuity of the functions below to evaluate the limits.

1. Let $f(x) = \begin{cases} \frac{1}{2}x + 4 & x < 0 \\ -1 & 0 \le x < 4 \\ x^2 - 8x + 15 & x \ge 4 \end{cases}$.

a. $\lim_{x \to -2} f(x) = 3$

b. $\lim_{x \to 3} f(x) = -1$

c. $\lim_{x \to 0^+} f(x) = -1$

d. $\lim_{x \to 0^-} f(x) = 4$

e. $\lim_{x \to 0} f(x) =$ DNE

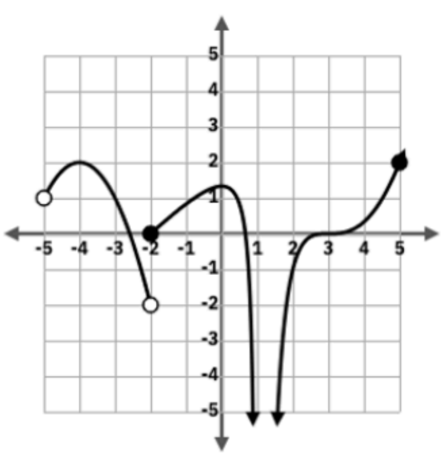
$\lim_{x \to 0^+} f(x) \neq \lim_{x \to 0^-} f(x)$

f. $\lim_{x \to 4^+} f(x) = -1$

g. $\lim_{x \to 4} f(x) = -1$

h. $\lim_{x \to 5} f(x) = 0$

2.



a. $\lim_{x \to -4} f(x) = 2$

b. $\lim_{x \to 3} f(x) = 0$

c. $\lim_{x \to -2^+} f(x) = 0$

d. $\lim_{x \to -2^-} f(x) = -2$

e. $\lim_{x \to 2} f(x) = -1$

f. $\lim_{x \to 5^-} f(x) = 2$

g. $\lim_{x \to 1^-} f(x) = -\infty$ (DNE)

h. $\lim_{x \to -5^+} f(x) = 1$