

LESSON 2.6: COMBINING FUNCTIONS

WARM
UPLet $f(x) = x^3 + 2x + 1$ and $g(x) = 2x - 7$

$$1. \text{ Find } f(x) - g(x) = (x^3 + 2x + 1) - (2x - 7) \\ = x^3 + 2x + 1 - 2x + 7 \\ = x^3 + 8$$

$$2. \text{ Find } f(x)g(x) = (x^3 + 2x + 1)(2x - 7) \\ = 2x^4 - 7x^3 + 4x^2 - 12x - 7$$

$$3. \text{ Find } f(2x - 7) = (2x - 7)^3 + 2(2x - 7) + 1$$

SUM,
DIFFERENCES,
PRODUCTS, AND
QUOTIENTS OF
functionsLet $f(x)$ and $g(x)$ be functions with domains A and B respectively.

Combination of Functions

Domain

$$(f + g)(x) = f(x) + g(x)$$

$$A \cap B$$

$$(f - g)(x) = f(x) - g(x)$$

$$A \cap B$$

$$(fg)(x) = f(x)g(x)$$

$$A \cap B$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

$$\{x \in A \cap B \mid g(x) \neq 0\}$$

Example: Let $f(x) = \frac{1}{x+3}$ and $g(x) = \sqrt{x}$

$$D: (-\infty, -3) \cup (-3, \infty) \quad D: [0, \infty)$$

a. Find the functions $f + g$, $f - g$, fg and $\frac{f}{g}$.

$$(f+g)(x) = \frac{1}{x+3} + \sqrt{x}$$

$$(fg)(x) = \frac{1}{x+3} \cdot \sqrt{x} = \frac{\sqrt{x}}{x+3}$$

$$(f-g)(x) = \frac{1}{x+3} - \sqrt{x}$$

$$\left(\frac{f}{g}\right)(x) = \frac{1}{x+3} \div \sqrt{x} = \frac{1}{x+3} \cdot \frac{1}{\sqrt{x}} = \frac{1}{\sqrt{x}(x+3)}$$

b. $(f + g)(4)$, $(f - g)(4)$, $(fg)(4)$ and $\left(\frac{f}{g}\right)(4)$.

$$(f+g)(4) = \frac{1}{4+3} + \sqrt{4} = \frac{1}{7} + 2 = \frac{15}{7}$$

$$(fg)(4) = \frac{\sqrt{4}}{4+3} = \frac{2}{7}$$

$$(f-g)(4) = \frac{1}{4+3} - \sqrt{4} = \frac{1}{7} - 2 = -\frac{13}{7}$$

$$\left(\frac{f}{g}\right)(4) = \frac{1}{\sqrt{4}(4+3)} = \frac{1}{2(7)} = \frac{1}{14}$$

We can create a composition of functions by inputting one function into another function.

$$f \circ g = f(g(x)) \quad \text{OR} \quad g \circ f = g(f(x))$$

Examples:

1. Let $f(x) = x^2$ and $g(x) = x - 2$.

a. Find the functions $f \circ g$ and $g \circ f$ and their domains.

$$f \circ g = f(g(x)) = f(x-2) = (x-2)^2 = x^2 - 4x + 4$$

$$D: (-\infty, \infty)$$

$$g \circ f = g(f(x)) = g(x^2) = (x^2) - 2 = x^2 - 2$$

$$D: (-\infty, \infty)$$

b. Find $(f \circ g)(5)$ and $(g \circ f)(4)$.

$$(f \circ g)(5) = (5)^2 - 4(5) + 4 = 25 - 20 + 4 = 9$$

$$(g \circ f)(4) = (4)^2 - 2 = 16 - 2 = 14$$

COMPOSITION
OF
functions

COMPOSITION
OF
functions

Examples:

2. If $f(x) = \sqrt{x}$ and $g(x) = \sqrt{4-x}$, find the following functions and their domains.

$$\begin{aligned} \text{a. } (f \circ g)(x) &= f(g(x)) \\ &= f(\sqrt{4-x}) \\ &= \sqrt{\sqrt{4-x}} \end{aligned}$$

$$D: (-\infty, 4]$$

$$\begin{aligned} \text{b. } (g \circ f)(x) &= g(f(x)) \\ &= g(\sqrt{x}) \\ &= \sqrt{4-\sqrt{x}} \end{aligned}$$

$$D: [0, 16]$$

$$\begin{aligned} \text{c. } (f \circ f)(x) &= f(f(x)) \\ &= f(\sqrt{x}) \\ &= \sqrt{\sqrt{x}} \end{aligned}$$

$$D: [0, \infty)$$

$$\begin{aligned} \text{d. } (g \circ g)(x) &= g(g(x)) \\ &= g(\sqrt{4-x}) \\ &= \sqrt{4-\sqrt{4-x}} \end{aligned}$$

$$D: [-12, 4]$$

3. If $f(x) = \frac{x}{x+1}$, $g(x) = x^4$, and $h(x) = x + 2$. Find $(f \circ g \circ h)(x)$.

$$(f \circ g \circ h)(x) = f(g(h(x))) = f((x+2)^4) = \frac{(x+2)^4}{(x+2)^4 + 1}$$

4. Given $F(x) = \sqrt[4]{2x-3}$, find functions f and g such that $F(x) = f \circ g$.

$$\text{Let } f(x) = \sqrt[4]{x} \quad \& \quad g(x) = 2x-3$$

$$\begin{aligned} f(g(x)) &= \sqrt[4]{2x-3} \\ &= F(x) \quad \checkmark \end{aligned}$$