ENTRY #17

Lesson 2.7: One-to-One Functions & Their Inverses

WARM UP

Determine which of the following sets are functions.

1.

x	0	4	-1	0	7
y	3	6	3	2	5

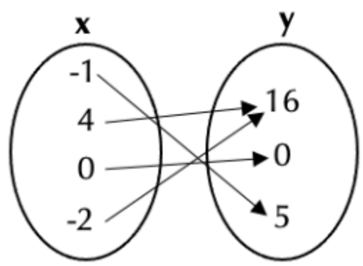
Not a function.

2.

x	-1	2	-1	6	0
y	4	5	6	4	2

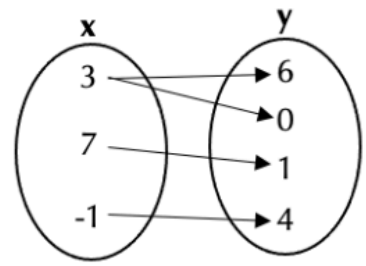
Not a function.

3.



Function ✓

4.



Not a function.

ONE-TO-ONE functions

A function is one-to-one if each input has a ___unique___ output.

In other words, no two inputs can have the same ___output___ .

One-To-One

x	1	-1	3	5	7	9
y	5	7	2	10	-3	0

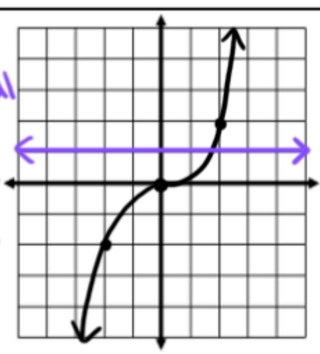
Not One-To-One *x=1 & x=3 have the same output

x	1	-1	3	5	7	9
y	5	7	5	10	-3	0

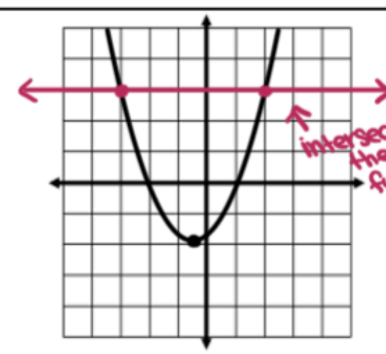
Graphically, we can test if a function is one-to-one by using the ___horizontal___ line test as demonstrated below.

One-To-One

*All horizontal lines will only intersect the function once.



Not One-To-One



intersects the function twice.

ONE-TO-ONE *functions*

Example: Which of the following functions are one-to-one?

1. $f(x) = x^2$

$(-3)^2 = 9$
$(3)^2 = 9$
$x=-3$ & $x=3$ has the same input.

Not one-to-one.

2. $f(x) = x^3$

$(\sqrt[3]{x_1})^3 = (\sqrt[3]{x_2})^3$

$x_1 = x_2$

$\Rightarrow$ Any number cubed has a unique output

$f(x)$ is one-to-one.

3. $f(x) = 2x - 4$

$2x_1 - 4 = 2x_2 - 4$
$+4 \quad +4$

$\frac{2x_1}{2} = \frac{2x_2}{2}$

$x_1 = x_2$

$f(x)$ is one-to-one.

INVERSE OF A *function*

The inverse of a one-to-one function is another <u>function</u>.

See the steps below to find the inverse of a one-to-one function.

Step 1	Write $y=f(x)$.
Step 2	Swap x and y.
Step 3	Solve for y. The resulting equation is $y=f^{-1}(x)$

Examples: Find the inverse of the functions below.

1. $f(x) = -2x + 1$

$y = -2x+1$

$x = -2y+1$
$-1 \quad -1$

$\frac{x-1}{-2} = \frac{-2y}{-2}$

$y = -\frac{1}{2}x + \frac{1}{2}$

$\boxed{f^{-1}(x) = -\frac{1}{2}x + \frac{1}{2}}$

2. $f(x) = \sqrt{x-4}$

$y = \sqrt{x-4}$

$(x)^2 = (\sqrt{y-4})^2$

$x^2 = y-4$
$+4 \quad +4$

$x^2+4 = y$

$\boxed{f^{-1}(x) = x^2+4}$

3. $f(x) = \frac{x^3-4}{2}$

$y = \frac{x^3-4}{2}$

$2(x) = \left(\frac{y^3-4}{2}\right)2$

$2x = y^3-4$
$+4 \quad +4$

$\sqrt[3]{2x+4} = \sqrt[3]{y^3}$

$y = \sqrt[3]{2x+4}$

$\boxed{f^{-1}(x) = \sqrt[3]{2x+4}}$

4. $f(x) = \frac{2x+3}{x-1}$

$y = \frac{2x+3}{x-1}$ Solve for x first!

$y(x-1) = 2x+3$

$yx - y = 2x+3$

$yx - 2x = y+3$

$\frac{x(y-2)}{y-2} = \frac{y+3}{y-2}$

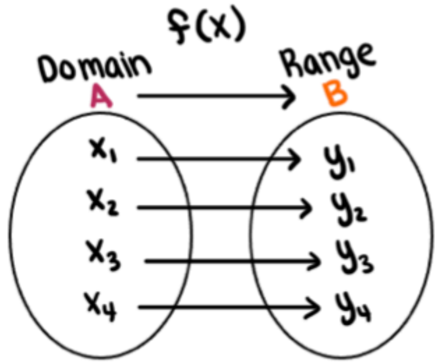
$x = \frac{y+3}{y-2}$

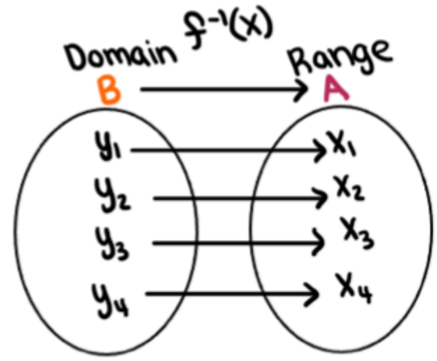
$y = \frac{x+3}{x-2}$

$\boxed{f^{-1}(x) = \frac{x+3}{x-2}}$

GRAPHING THE INVERSE OF A *function*

Since we switch the input with the output in order to find the inverse of a one-to-one function, the original function's domain becomes the inverse function's <u>range</u> and the original function's range becomes the inverse function's <u>domain</u> .





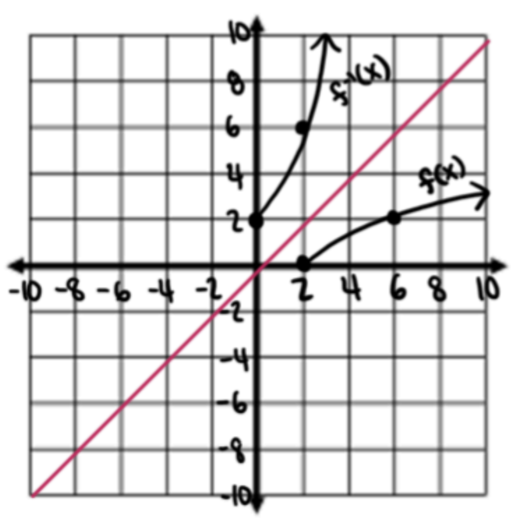
As a result, we can graph the inverse of a function by switching each x coordinate with the corresponding y coordinate of the original function.

For example, if the point $(3, -2)$ is on $f(x)$ then, <u>(-2,3)</u> is on $f^{-1}(x)$.

To achieve this, we could also take $f(x)$ and reflect it over the line $y = x$ to obtain the graph of $f^{-1}(x)$.

Examples: Graph the inverse of each of the functions below.

1. $f(x) = \sqrt{x - 2}$



2. $f(x) = x^3$