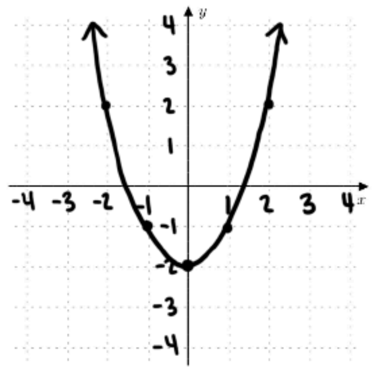
LESSON 3.1: QUADRATIC FUNCTIONS

WARM UP

Fill in the table and graph the functions below.

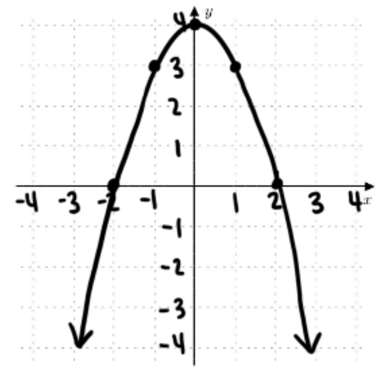
1. $y = x^2 - 2$

x	-3	-2	-1	0	1	2	3
y	7	2	-1	-2	-1	2	7



2. $y = -x^2 + 4$

x	-3	-2	-1	0	1	2	3
y	-5	0	3	4	3	0	-5



VERTEX (STANDARD) FORM OF QUADRATIC *functions*

A quadratic function is a polynomial function of degree 2. It is of the form:

$$f(x) = ax^2 + bx + c$$

We can use the process of completing the square to rewrite the equation of a quadratic function in vertex (or standard) form:

$$f(x) = a(x-h) + k \quad \text{where } (h,k) \text{ is the vertex.}$$

Examples: Put the quadratic equation in standard form. Then, state the vertex.

1. $f(x) = x^2 + 4x - 1$

$x^2+4x-1=0$
$\quad +1 \quad +1$
$x^2+4x+\underline{4} = 1+\underline{4}$
$(x+2)^2 = 5$
$\quad -5 \quad -5$
$(x+2)^2 - 5 = 0$

$f(x) = (x+2)^2 - 5$

vertex: $(-2,-5)$

2. $f(x) = 3x^2 - 6x + 2$

$3x^2 - 6x + 2 = 0$
$\quad -2 \quad -2$
$3(x^2 - 2x + \underline{1}) = -2 + 3(\underline{1})$
$3(x-1)^2 = 1$
$\quad -1 \quad -1$
$3(x-1)^2 - 1 = 0$

$f(x) = (x-1)^2 - 1$

vertex: $(1,-1)$

3. $f(x) = x^2 + 3x + 12$

$x^2 + 3x + 12 = 0$
$\quad -12 \quad -12$
$x^2 + 3x + \underline{9/4} = -12 + \underline{9/4}$
$(x+3/2)^2 = -48/4 + 9/4$
$(x+3/2)^2 = -39/4$
$(x+3/2)^2 + 39/4 = 0$

$f(x) = (x+\frac{3}{2})^2 + \frac{39}{4}$

vertex: $(-\frac{3}{2}, \frac{39}{4})$

4. $f(x) = -x^2 + 2x - 7$

$-x^2 + 2x - 7 = 0$
$\quad +7 \quad +7$
$-(x^2 - 2x + \underline{1}) = 7 + (-1)(\underline{1})$
$-(x-1)^2 = 6$
$-(x-1)^2 - 6 = 0$

$f(x) = -(x-1)^2 - 6$

vertex: $(1,-6)$

GRAPHING QUADRATIC functions

We can use the standard or vertex form of a quadratic function to help us graph them.

The standard or vertex form of a quadratic function tells us how the parent function $y = x^2$ is being shifted :

shift left/right

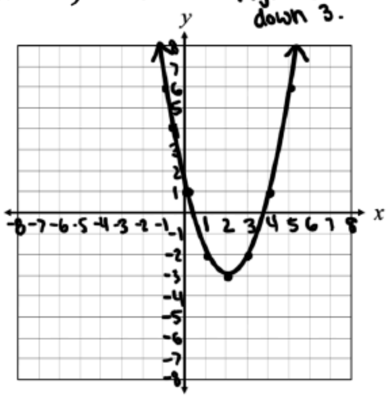
$f(x) = a(x-h)^2 + k$ ← shift up/down

vertical squeeze/ stretch OR reflection over x-axis

Examples: Put each of the quadratic functions below in vertex form, then graph them.

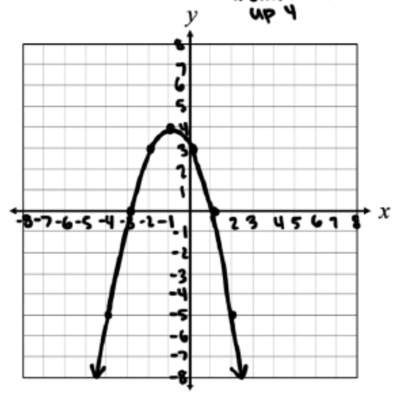
1. $f(x) = (x-2)^2 - 3$

*shift $y=x^2$ right 2 and down 3.



2. $f(x) = -(x+1)^2 + 4$

*reflect over y-axis
*shift left 1 & up 4



3. $f(x) = x^2 + 6x + 1$

$x^2+6x+1=0$
$\quad\;\; -1 \quad -1$

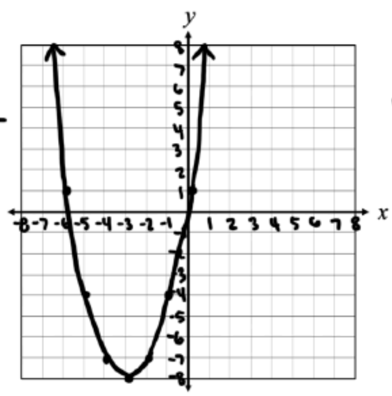
$x^2+6x+\underline{9} = -1+\underline{9}$

$(x+3)^2 = 8$

$(x+3)^2 - 8 = 0$

$f(x) = (x+3)^2 - 8$

*shift left 3 and down 8



4. $f(x) = 5x^2 - 10x + 20$

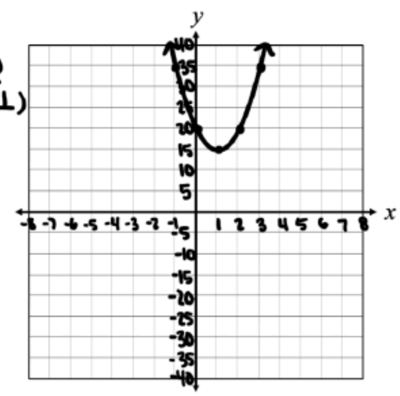
$5x^2-10x+20=0$
$\quad\quad -20 \quad -20$

$5(x^2-2x+\underline{1}) = -20+5(\underline{1})$

$5(x-1)^2 = -15$

$5(x-1)^2+15=0$

$f(x)=5(x-1)^2+15$



5. $f(x) = -x^2 - 8x - 5$

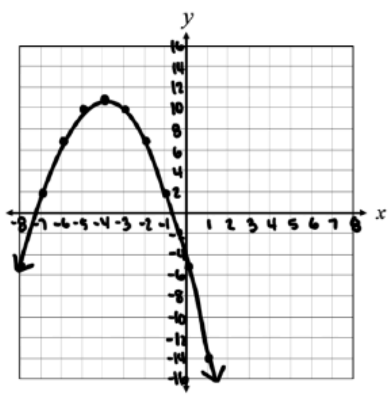
$-x^2-8x-5=0$
$\quad\quad +5 \quad +5$

$-(x^2+8x+\underline{16}) = 5+(-1)(\underline{16})$

$-(x+4)^2 = -11$

$-(x+4)^2+11=0$

$f(x) = -(x+4)^2+11$



6. $f(x) = -2x^2 + 8x + 3$

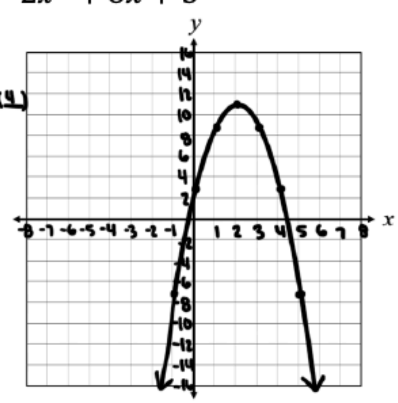
$-2x^2+8x+3=0$
$\quad\quad -3 \quad -3$

$-2(x^2-4x+\underline{4}) = -3+(-2)(\underline{4})$

$-2(x-2)^2 = -11$

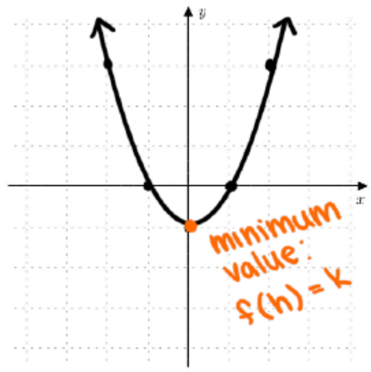
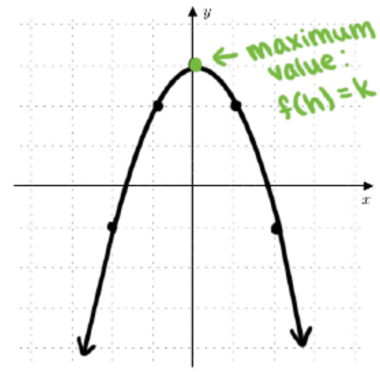
$-2(x-2)^2+11=0$

$f(x) = -2(x-2)^2+11$



MAXIMUMS AND MINIMUMS OF QUADRATIC *functions*

The maximum or minimum value of a quadratic function occurs at the __vertex__.

When $a > 0$, a quadratic function has a __minimum__ at the __vertex__.	When $a < 0$, a quadratic function has a __maximum__ at the __vertex__.
	

Examples: Do the functions below have a maximum or minimum? Find the value.

1. $f(x) = -2x^2 + 4x - 5$

$a = -2 < 0 \Rightarrow f$ has a maximum

$-2x^2 + 4x - 5 = 0$

$\quad +5 \; +5$

$-2(x^2 - 2x + \underline{1}) = 5 + (-2)(\underline{1})$

$-2(x-1)^2 = 3$

$\quad -3 \; -3$

$-2(x-1)^2 - 3 = 0$

$f(x) = -2(x-1)^2 - 3$

vertex: $(1, -3)$ Max: $f(1) = -3$

2. $f(x) = x^2 + 10x - 5$

$a = 1 > 0 \Rightarrow f(x)$ has a minimum

$x^2 + 10x - 5 = 0$

$\quad +5 \; +5$

$x^2 + 10x + \underline{25} = 5 + \underline{25}$

$(x+5)^2 = 30$

$\quad -30 \; -30$

$(x+5)^2 - 30 = 0$

$f(x) = (x+5)^2 - 30$

vertex: $(-5, -30)$

Minimum: $f(-5) = -30$

3. $f(x) = x^2 + 8x$

$a = 1 > 0 \Rightarrow f(x)$ has a minimum

$x^2 + 8x = 0$

$x^2 + 8x + \underline{16} = 0 + \underline{16}$

$(x+4)^2 = 16$

$\quad -16 \; -16$

$(x+4)^2 - 16 = 0$

$f(x) = (x+4)^2 - 16$

vertex: $(-4, -16)$

Minimum: $f(-4) = -16$

4. $f(x) = -x^2 - 3x + 4$

$a = -1 < 0 \Rightarrow f(x)$ has a maximum

$-x^2 - 3x + 4 = 0$

$\quad -4 \; -4$

$-(x^2 + 3x + \underline{9/4}) = -4 + (-1)(\underline{9/4})$

$-(x + 3/2)^2 = -\frac{16}{4} - \frac{9}{4}$

$-(x + 3/2)^2 = -25/4$

$-(x + 3/2)^2 + 25/4 = 0$

$f(x) = -(x + \frac{3}{2})^2 + 25/4$

Vertex: $(-3/2, 25/4)$

Maximum: $f(-3/2) = 25/4$