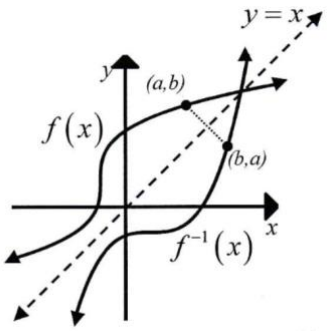
Entry #: _____

Lesson 3.2: Derivatives of Inverse Functions

Review of Inverse Functions



At right are graphs of a function $f(x)$ and its inverse $f^{-1}(x)$.

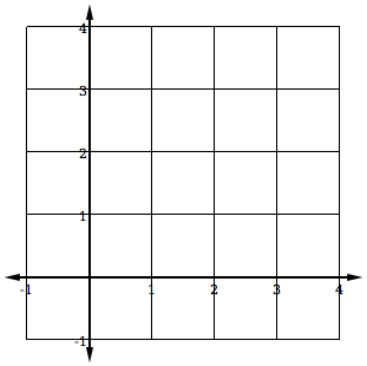
If the graph of f contains the point (a, b), then the graph of f^{-1} contains the point _____ .

Also, the graph of f^{-1} is the _____ of the graph of f over the line _____ .

From the graphs to the right, what do you notice about the relationship between the slope of the graph of f at (a, b) and the slope of the graph of f^{-1} at the point (b, a)?

Examples:

1. Let $f(x) = \sqrt{x}$.
   a. Sketch the graph of $f(x)$.
   b. Find $f^{-1}(x)$. Hint: You must list a domain restriction.



   c. Sketch the graph of $f^{-1}(x)$ on the same coordinate plane.
   d. Differentiate both $f(x)$ and $f^{-1}(x)$.

   e. Find the slope of the graph of $f(x)$ at $(4,2)$ and the slope of the graph of $f^{-1}(x)$ at $(2,4)$.

   f. What conclusion can you make about these slopes?

Since $slope = m = \frac{\Delta y}{\Delta x}$, it should make sense that switching x and y (for inverse functions)

should produce _____ for inverse functions.

Entry #: _____

Derivatives of Inverse Functions

If a function f' has an inverse function f^{-1}, then f is one-to-one and must be either strictly increasing or strictly decreasing (strictly monotonic) on its entire domain.

We can use f' to find out where f is increasing and where f is decreasing.

Examples:

1. Let f and g be inverse functions such that:

   $f(-1) = 1$ $\quad$ $f(0) = 2$ $\quad$ $f(1) = 5$

   $f'(-1) = \frac{3}{2}$ $\quad$ $f'(0) = 2$ $\quad$ $f'(1) = \frac{1}{2}$

   From the given information, find each of the following, if possible.
   Hint: Make a table or chart to organize your data.

   a. $g'(1)$ $\quad$ b. $g'(2)$ $\quad$ c. $g'(3)$ $\quad$ d. $g'(0)$ $\quad$ e. $g'(5)$

2. a. Use $f'(x)$ to show that $f(x) = 6x - x^3$ is not one-to-one on its entire domain.

   b. Find the largest interval containing $x = 0$ for which f is one-to-one.

   c. Find the largest interval containing $x = -2$ for which f has an inverse function.