Lesson 33: Dividing Polynomials

WARM UP

Divide using long division. State the remainder if necessary.

1. $365 \div 5 = 73$

$$\begin{array}{r} 73 \\ 5\overline{)365} \\ -35\downarrow \\ \hline 15 \\ -15 \\ \hline 0 \end{array}$$

$\leftarrow$ no remainder

2. $421 \div 12$

$$\begin{array}{r} 35 \text{ R.1} \\ 12\overline{)421} \\ -36\downarrow \\ \hline 61 \\ -60 \\ \hline 1 \end{array}$$

$$\frac{421}{12} = 35 + \frac{1}{12}$$

THE DIVISION ALGORITHM FOR *polynomials*

Let's review a little bit of vocabulary:

$$\underbrace{\frac{\boxed{38}}{\boxed{7}}}_{\text{divisor}}^{\text{dividend}} = \underbrace{\boxed{5}}_{\text{quotient}} + \frac{\boxed{3}\leftarrow\text{remainder}}{7}$$

We use the same vocabulary when dividing polynomials:

$$\underbrace{\frac{\boxed{P(x)}}{\boxed{D(x)}}}_{\text{divisor}}^{\text{dividend}} = \underbrace{\boxed{Q(x)}}_{\text{quotient}} + \frac{\boxed{R(x)}\leftarrow\text{remainder}}{D(x)}$$

Multiplying both sides by the quotient, we end up with our original polynomial:

$$P(x) = Q(x) \cdot D(x) + R(x)$$

Example: Identify the dividend, quotient, divisor and remainder below.

$$\underbrace{\frac{\boxed{6x^2 - 26x + 12}}{\boxed{x - 4}}}_{\text{divisor}}^{\text{dividend}} = \underbrace{\boxed{6x - 2}}_{\text{quotient}} + \frac{\boxed{4}\leftarrow\text{remainder}}{x - 4}$$

$$6x^2 - 26x + 12 = (6x - 2)(x - 4) + 4$$

LONG DIVISION OF *polynomials*

Dividing polynomials is a very similar process to using long division to divide numbers. Express your answer in the form $\frac{P(x)}{D(x)} = Q(x) + \frac{R(x)}{D(x)}$.

Examples:

1. $\frac{x^3+2x^2-5x-6}{x-2}$

x^2+4x+3 R.0

$x-2 \overline{)\, x^3+2x^2-5x-6}$

$-(x^3-2x^2)$

$4x^2-5x$

$-(4x^2-8x)$

$3x-6$

$-(3x-6)$

$\emptyset \leftarrow$ no remainder

$$\boxed{\frac{x^3+2x^2-5x-6}{x-2} = x^2+4x+3}$$

2. $\frac{3x^5+5x^4-4x^3+7x+3}{x+2}$

$3x^4-x^3-2x^2+4x-1$ R.5

$x+2 \overline{)\, 3x^5+5x^4-4x^3+7x+3}$

$-(3x^5+6x^4)$

$-x^4-4x^3$

$-(-x^4-2x^3)$

$-2x^3+7x$

$-(-2x^3-4x^2)$

$4x^2+7x$

$-(4x^2+8x)$

$-x+3$

$-(-x-2)$

5

$$\boxed{\frac{3x^5+5x^4-4x^3+7x+3}{x+2} = 3x^4-x^3-2x^2+4x-1+\frac{5}{x+2}}$$

3. $\frac{8x^4+6x^2-3x+1}{2x^2-x+2}$

$4x^2+2x$ R.$-7x+1$

$2x^2-x+2 \overline{)\, 8x^4+6x^2-3x+1}$

$-(8x^4-4x^3+8x^2)$

$4x^3-2x^2-3x$

$-(4x^3-2x^2+4x)$

$-7x+1$

$$\boxed{\frac{8x^4+6x^2-3x+1}{2x^2-x+2} = 4x^2+2x+\frac{-7x+1}{2x^2-x+2}}$$

4. $\frac{2x^3-7x^2+5}{x-3}$

$2x^2-x-3$ R:4

$x-3 \overline{)\, 2x^3-7x^2+5}$

$-(2x^3-6x^2)$

$-x^2+5$

$-(-x^2+3x)$

$-3x+5$

$-(-3x+9)$

-4

$$\boxed{\frac{2x^3-7x^2+5}{x-3} = 2x^2-x-3+\frac{-4}{x-3}}$$

SYNTHETIC DIVISION OF *polynomials*

Synthetic division is a great method to use if the divisor of the form $x - c$.

Examples: Use synthetic division to divide the polynomials below.

1. $\frac{x^3+2x^2-5x-6}{x-2}$

$x-2 \Rightarrow c=2$

$$\begin{array}{r|rrrr} 2 & 1 & 2 & -5 & -6 \\ & \downarrow & 2 & 8 & 6 \\ \hline & 1 & 4 & 3 & \emptyset \end{array}$$

x^2+4x+3 ← remainder

$$\boxed{\frac{x^3+2x^2-5x-6}{x-2} = x^2+4x+3}$$

2. $\frac{3x^5+5x^4-4x^3+7x+3}{x+2}$

$x+2 \Rightarrow C=-2$

$$\begin{array}{r|rrrrrr} -2 & 3 & 5 & -4 & 0 & 7 & 3 \\ & \downarrow & -6 & 2 & 4 & -8 & 2 \\ \hline & 3 & -1 & -2 & 4 & -1 & \boxed{5} \end{array}$$

$x^4-x^3-2x^2+4x-1$ ← remainder

$$\boxed{\frac{3x^5+5x^4-4x^3+7x+3}{x+2} = x^4-x^3-2x^2+4x-1+\frac{5}{x+2}}$$

3. $\frac{8x^4+6x^2-3x+1}{x+1}$

$x+1 \Rightarrow c=-1$

$$\begin{array}{r|rrrrr} -1 & 8 & 0 & 6 & -3 & 1 \\ & \downarrow & -8 & 8 & -14 & 17 \\ \hline & 8 & -8 & 14 & -17 & \boxed{18} \end{array}$$

← remainder

$8x^3-8x^2+14x-17$

$$\boxed{\frac{8x^4+6x^2-3x+1}{x+1} = 8x^3-8x^2+14x-17+\frac{18}{x+1}}$$

4. $\frac{2x^3-7x^2+5}{x-3}$

$x-3 \Rightarrow C=3$

$$\begin{array}{r|rrrr} 3 & 2 & -7 & 0 & 5 \\ & \downarrow & 6 & -3 & -9 \\ \hline & 2 & -1 & -3 & \boxed{-4} \end{array}$$

← remainder

$2x^2-x-3$

$$\boxed{\frac{2x^3-7x^2+5}{x-3} = 2x^2-x-3+\frac{-4}{x-3}}$$

THE REMAINDER Theorem

If the polynomial $P(x)$ is divided by $x - c$, then the remainder is the value $P(c)$.

If $P(c) = 0$, then $x - c$ is a factor of $P(x)$.

Example: Use the remainder theorem to find the remainder when $P(x) = 3x^5 + 5x^4 - 4x^3 + 7x + 3$ is divided by $x + 2$. Does it confirm your answer from #2 in the previous examples?

$x+2$

$\Rightarrow c = -2$

$$\begin{aligned} P(-2) &= 3(-2)^5 + 5(-2)^4 - 4(-2)^3 + 7(-2) + 3 \\ &= 3(-32) + 5(16) - 4(-8) - 14 + 3 \\ &= -96 + 80 + 32 - 14 + 3 \\ &= -16 + 32 - 11 \\ &= 16 - 11 \\ &= 5 \end{aligned}$$

yes, it is the same remainder.

THE FACTOR Theorem

c is a __zero__ of $P(x)$ if and only if $x - c$ is a __factor__ of $P(x)$.

Example: Let $P(x) = x^3 - 7x + 6$. Show that $P(1) = 0$, and use this fact to factor $P(x)$ completely.

$$\begin{aligned} P(1) &= (1)^3 - 7(1) + 6 \\ &= 1 - 7 + 6 \\ &= -6 + 6 \\ &= 0 \checkmark \end{aligned}$$

$\Rightarrow 1$ is a zero of $P(x)$

$\Rightarrow x-1$ is a factor of $P(x)$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & -7 & 6 \\ & \downarrow & 1 & 1 & -6 \\ \hline & 1 & 1 & -6 & 0 \end{array}$$

$x^2 + x - 6$

$P(x) = (x-1)(x^2 + x - 6)$

$$\boxed{P(x) = (x-1)(x+3)(x-2)}$$