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# **Lesson 3.4:** Real Zeros of Polynomials

Rational Zeros of Polynomials

**Rational Zeros Theorem:** If the polynomial $P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ has integer coefficients, then every rational zero of $P$ is of the form:

Example: $P(x) = x^3 - 3x + 2$

How to Find the Rational Zeros of a Polynomial

1. List all possible _____________ using the _________________________ theorem.
2. Use __________________________ or ________________________________________ to evaluate the polynomial at each of the candidates for the rational zeros that you found in Step 1. When the remainder is 0, note the quotient you have obtained.
3. Repeat steps 1 and 2 until you reach a quotient that is a quadratic or factors easily. Use the quadratic formula or factor to find the remaining zeros.

Example: $P(x) = 2x^3 + x^2 - 13x + 6$

Descartes' Rule of Signs

Let $P$ be a polynomial with real coefficients.

1. The number of ______________________ zeros of $P(x)$ either is equal to the number of variations in sign in $P(x)$ or is less than that by an even whole number.

2. The number of _______________________ zeros of $P(x)$ is either equal to the number of variations in sign in $P(-x)$ or less than that by an even whole number.

Example: Use Descartes' Rule to determine the possible number of positive and negative real zeros of the polynomial $P(x) = 3x^6 + 4x^5 + 3x^3 - x - 3$.

The Upper and Lower Bounds Theorem

Let $P$ be a polynomial with real coefficients.

1. If we divide $P(x)$ by $x - b$ (with $b > 0$) using synthetic division and if the row that contains the quotient and remainder has no negative entry, then $b$ is an upper bound for the real zeros of $P$.

2. If we divide $P(x)$ by $x - a$ (with $a < 0$) using synthetic division and if the row that contains the quotient and the remainder has entries that are alternately nonpositive and nonnegative, then $a$ is a lower bound for the real zeros of $P$.

Example: Show that all the real zeros of the polynomial $P(x) = x^4 - 3x^2 + 2x - 5$ lie between -3 and 2.