ENTRY #22

Lesson 3.5: Complex Numbers

WARM UP

Solve the equations below.

1. $x^2 - 1 = 0$

$$\begin{aligned} &+1 \quad +1 \\ &\sqrt{x^2} = \sqrt{1} \\ &x = \pm 1 \end{aligned}$$

2. $x^2 + 4 = 0$

$$\begin{aligned} &-4 \quad -4 \\ &\sqrt{x^2} = \sqrt{-4} \end{aligned}$$

no real solution

WHAT ARE complex NUMBERS?

A complex number is an expression of the form:

$a + bi$

"real" part (a) "imaginary" part (b)

* where $i = \sqrt{-1}$

Complex Number	Real Part	Imaginary Part
$3+4i$	3	4
$\frac{1}{2} - \frac{2}{3}i$	$\frac{1}{2}$	$-\frac{2}{3}$
$6i$	0	6
-7	-7	0

ARITHMETIC OPERATIONS ON COMPLEX numbers

Addition	$(a+bi)+(c+di)=(a+c)+(b+d)i$
Subtraction	$(a+bi)-(c+di)=(a-c)+(b-d)i$
Multiplication FOIL!	$(a+bi)(c+di)=(a)(c)+(a)(di)+(bi)(c)+\boxed{(bi)(di)}$ $\quad (i)^2=-1$

Examples: Express the following in the form $a + bi$.

1. $(2+4i)+(3-3i)$
$= (2+3)+(4+(-3))i$
$= 5+i$

2. $(4+5i)-(2-4i)$
$= (4-2)+(5-(-4))i$
$= 2+9i$

3. $(1+3i)(2-2i)$
$= (1)(2)+(1)(-2i)+(3i)(2)+(3i)(-2i)$
$= 2-2i+6i-6i^2$
$= 2+4i-6(-1)$
$= 8+4i$

4. $i^{25} = i^{24+1} = i^{24} i$
$= (i^2)^{12} i$
$= (-1)^{12} i$
$= i$

DIVIDING COMPLEX *numbers*

To simplify the quotient $\frac{a+bi}{c+di}$, multiply the numerator and denominator by the complex conjugate of the denominator:

$(c+di \leftrightarrow c-di)$

$$\frac{a+bi}{c+di}\cdot\frac{(c-di)}{(c-di)}$$

* multiply through, then simplify.

Examples: Express the following in the form $a+bi$.

1. $\frac{(2+4i)}{(1-3i)}\cdot\frac{(1+3i)}{(1+3i)} = \frac{(2)(1)+(2)(3i)+(4i)(1)+(4i)(3i)}{(1)^2-(3i)^2}$
   $= \frac{2+6i+4i+12i^2}{1-9i^2}$
   $= \frac{2+10i-12}{1+9}$
   $= \frac{-10+10i}{10}$
   $= -1+i$

2. $\frac{(5+2i)}{(3i)}\cdot\frac{(-3i)}{(-3i)} = \frac{-15i-6i^2}{-9i^2}$
   $= \frac{-15i-6(-1)}{-9(-1)}$
   $= \frac{6-15i}{9}$
   $= \frac{2}{3}-\frac{5}{3}i$

SQUARE ROOTS OF NEGATIVE *numbers*

$\sqrt{-r} = \sqrt{-1}\sqrt{r} = i\sqrt{r}$
*evaluating

$\sqrt{x^2} = \sqrt{-r} \Rightarrow x = \pm\sqrt{-r}$ *solving
$x = -i\sqrt{r}$ $x = i\sqrt{r}$

Examples:

1. Take the square roots of the numbers below.
   a. $\sqrt{-1} = i$
   b. $\sqrt{-16} = \sqrt{-1}\sqrt{16} = i4 = 4i$
   c. $\sqrt{-3} = \sqrt{-1}\sqrt{3} = i\sqrt{3}$
2. Evaluate $(\sqrt{12}-\sqrt{-3})(3+\sqrt{-4})$ and express it in $a+bi$ form.
   $(2\sqrt{3}-i\sqrt{3})(3+2i) = (2\sqrt{3})(3)+(2\sqrt{3})(2i)-(i\sqrt{3})(3)-(i\sqrt{3})(2i)$
   $= 6\sqrt{3}+4i\sqrt{3}-3i\sqrt{3}+2\sqrt{3}$
   $= 8\sqrt{3}+i\sqrt{3}$

COMPLEX SOLUTIONS OF QUADRATIC *equations*

Examples: Solve the equations below for all complex solutions.

1. $x^2+16=0$
   $-16 \quad -16$
   $\sqrt{x^2} = \sqrt{-16}$
   $x = \pm\sqrt{-16}$
   $x = \pm 4i$
   $\boxed{x=4i \quad x=-4i}$

2. $x^2+4x+5=0$
   $x = \frac{-(4)\pm\sqrt{(4)^2-4(1)(5)}}{2(1)}$
   $x = \frac{-4\pm\sqrt{16-20}}{2}$
   $x = \frac{-4\pm\sqrt{-4}}{2}$
   $x = \frac{-4\pm 2i}{2}$
   $x = -2\pm i$
   $\boxed{x=-2+i \quad x=-2-i}$