Entry #: 16

Lesson 4.3: Approximating with the Tangent Line

Sometimes, finding the value of a function is difficult or even impossible.

With the use of Calculus techniques, we can approximate the function value by finding a y-value on a line that is tangent to the function.

Since this method involves using a linear function (the tangent line function) at a nearby point, it is sometimes called local linearization approximation.

Examples:

1. If $(2, -2)$ is a point on the graph of $x^2 + y^2 + 2y = 4$, use the equation of a tangent line passing through the point $(2, -2)$ to approximate a y-coordinate:

   a. when the x-coordinate is 2.1

   $\frac{d}{dx}(x^2 + y^2 + 2y) = \frac{d}{dx}(4)$

   $2x + 2y\frac{dy}{dx} + 2\frac{dy}{dx} = 0$

   $-2x \qquad -2x$

   $2y\frac{dy}{dx} + 2\frac{dy}{dx} = -2x$

   $\frac{dy}{dx}(2y+2) = -2x$

   $\frac{dy}{dx} = \frac{-2x}{2y+2}$

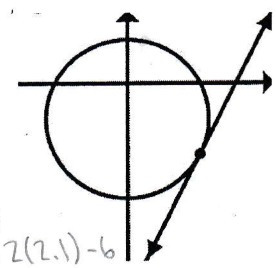
   $m_{tan} = \frac{-2(2)}{2(-2)+2}$

   $= 2$

   $y + 2 = 2(x-2)$

   $y = 2x - 6 \rightarrow y \approx 2(2.1) - 6$

   $\Rightarrow y \approx -1.8$

   b. when the x-coordinate is 1.9

   $y \approx 2(1.9) - 6$

   $y \approx -2.2$

2. If $f(2) = 3$ and $f'(2) = -2$, use local linearization to approximate $f(2.01)$.

   $(2, 3)$

   $y - y_1 = m(x - x_1)$

   $y - 3 = -2(x - 2)$

   $y = -2x + 7$

   $f(2.01) \approx -2(2.01) + 7$

   $f(2.01) \approx 2.98$