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Lesson 4.4: Laws of Logarithms

Laws of Logarithms

Let a be a positive number, with $a \neq 1$. Let A, B, and C be any real numbers with A > 0 & B > 0.

$\log_a(AB) = \log_a(A) + \log_a(B)$
$\log_a\left(\frac{A}{B}\right) = \log_a(A) - \log_a(B)$
$\log_a(A^C) = C\log_a(A)$

Examples: Evaluate each expression using the laws of logarithms.

1. $\log_4 2 + \log_4 32 =$ $\log_4(2\cdot 32) = \log_4(64) = \log_4(4^3) = 3$

2. $\log_2 80 - \log_2 5 =$ $\log_2\left(\frac{80}{5}\right) = \log_2(16) = \log_2(2^4) = 4$

3. $-\frac{1}{3}\log 8 =$ $\log\left(8^{-1/3}\right) = \log\left(\frac{1}{8}\right)^{1/3} = \log\left(\frac{1}{2}\right)$

Examples: Use laws of logarithms to expand each expression.

1. $\log_2(6x) =$ $\log_2(6) + \log_2(x)$

2. $\log_5(x^3y^6) =$ $\log_5(x^3) + \log_5(y^6) = 3\log_5(x) + 6\log_5(y)$

3. $\ln\left(\frac{ab}{\sqrt[3]{c}}\right) =$ $\ln(ab) - \ln\left(\sqrt[3]{c}\right) = \ln(a) + \ln(b) - \ln\left(c^{1/3}\right)$
$= \ln(a) + \ln(b) - \frac{1}{3}\ln(c)$

Examples: Use laws of logarithms to condense each expression into a single logarithm.

1. $3\log(x) + \frac{1}{2}\log(x+1) =$ $\log(x^3) + \log(x+1)^{1/2}$

$= \log\left(x^3(x+1)^{1/2}\right)$

2. $3\ln(s) + \frac{1}{2}\ln(t) - 4\ln(t^2+1) =$ $\ln(s^3) + \ln(t^{1/2}) - \ln(t^2+1)^4$

$= \ln(s^3 t^{1/2}) - \ln(t^2+1)^4$

$= \ln\left(\frac{s^3 t^{1/2}}{(t^2+1)^4}\right)$

Change of Base Formula

$$\log_b(x) = \frac{\log_a(x)}{\log_a(b)}$$

Example: Use the change of base formula, so that you can evaluate the following logarithm with a calculator (correct to 5 decimal places).

$\log_8 5$

$$\log_8(5) = \frac{\log(5)}{\log(8)} \approx 0.77398$$