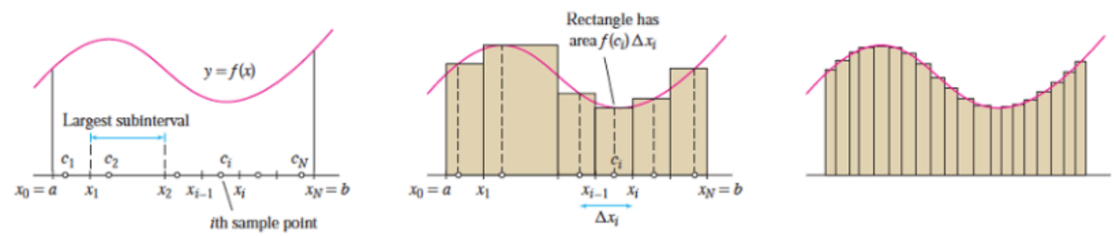
ENTRY #27

Lesson 5.2: The Definite Integral

RIEMANN Sums	Last lesson, we learned about how to approximate the area below a curve over an interval by creating _N__ rectangles of height _$f(x_i)$_ and width _Δx__ and adding up their areas. Note, all rectangles had the same _width__ in the previous methods. Riemann Sums are similar; however, not all rectangles need to have equal widths.



(A) Partition of $[a, b]$ into subintervals (B) Construct rectangle above each subinterval of height $f(c_i)$ (C) Rectangles corresponding to a Riemann sum with $\|P\|$ small (a large number of rectangles)

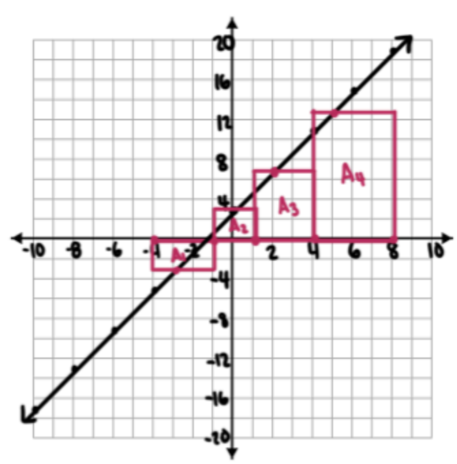
$$R(f, P, C) = \Delta x_1 (f(C_1)) + \Delta x_2 (f(C_2)) + \cdots + \Delta x_n f(C_n)$$

f(x) partition (sub-intervals) location of height on sub-interval

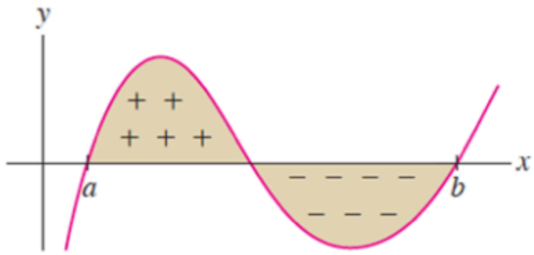
Example: Calculate the Riemann Sum $R(f,P,C)$ for the given function, partition and choice of sample points. Also, sketch the graph of f and the rectangles corresponding to $R(f,P,C)$.

$$f(x) = 2x + 3 \qquad P = \{-4,-1,1,4,8\} \qquad C = \{-3,0,2,5\}$$

$R(f,P,C) = (3)(f(-3)) + (2)(f(0)) + (3)(f(2)) + (4)(f(5))$
$= (3)(-3) + (2)(3) + (3)(7) + 4(13)$
$= -9 + 6 + 21 + 52$
$= 70$



SIGNED area



$$\int_a^b f(x)\,dx = \begin{pmatrix}\text{positive}\\ \text{area}\end{pmatrix} - \begin{pmatrix}\text{negative}\\ \text{area}\end{pmatrix}$$

area above x-axis

area below x-axis

Example: Calculate the integrals below by graphing and using geometry.

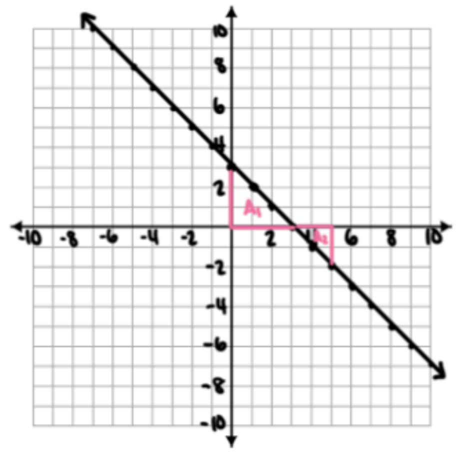
1. $\int_0^5 (3-x)dx$

$= A_1 - A_2$

$= \frac{1}{2}(3)(3) - \frac{1}{2}(2)(2)$

$= \frac{9}{2} - \frac{4}{2}$

$= \frac{5}{2}$

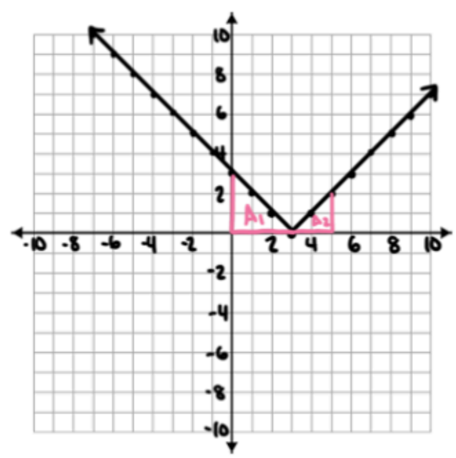


2. $\int_0^5 |3-x|\,dx = A_1 + A_2$

$= \frac{1}{2}(3)(3) + \frac{1}{2}(2)(2)$

$= \frac{9}{2} + \frac{4}{2}$

$= \frac{13}{2}$



PROPERTIES OF DEFINITE *integrals*

$\int_a^b C\,dx = C(b-a)$	$\int_a^b (f(x)+g(x))\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$
$\int_a^b C\,f(x)\,dx = C\int_a^b f(x)\,dx$	$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$
$\int_a^a f(x) = 0$	$\int_0^b x\,dx = \frac{1}{2}(b)^2$
$\int_0^b x^2\,dx = \frac{1}{3}(b)^3$	$\int_0^b x^3\,dx = \frac{1}{4}(b)^4$

Examples: Use the properties above to evaluate the integrals below.

1. $\int_0^3 (2x^2 - 5)\,dx$

$= 2\int_0^3 x^2 dx - \int_0^3 5\,dx$

$= 2\left(\frac{1}{3}(3)^3\right) - (3-0)(5)$

$= 2(9) - (3)(5)$

$= 18 - 15$

$= 3$

2. If $\int_5^8 f(x)\,dx = -12$, find $\int_8^5 f(x)\,dx$.

$\int_8^5 f(x)dx = -\int_5^8 f(x)\,dx$

$= -(-12)$

$= 12$

3. $\int_4^7 x^2\,dx = \int_0^7 x^2 dx - \int_0^4 x^2\,dx$

$= \frac{1}{3}(7)^3 - \frac{1}{3}(4)^3$

$= 93$

4. $\int_0^4 (t^2 + 6t - 1)\,dt$

$= \int_0^4 t^2 dt + 6\int_0^4 t\,dt - \int_0^4 1\,dt$

$= \frac{1}{3}(4)^3 + 6\left(\frac{1}{2}(4)^2\right) - 4$

$= \frac{64}{3} + 48 - 4$

$= \frac{64}{3} + 44$

$= \frac{64}{3} + \frac{132}{3}$

$= \frac{196}{3}$

5. $\int_{-2}^2 (2x^3 - 3x^2)\,dx = \int_{-2}^0 (2x^3-3x^2)dx + \int_0^2 (2x^3-3x^2)dx$

$= -\int_0^{-2}(2x^3-3x^2)dx + 2\int_0^2 x^3 dx - 3\int_0^2 x^2 dx$

$= -2\int_0^{-2} x^3 + 3\int_0^{-2} x^2 dx + 2\left(\frac{1}{4}(2)^4\right) - 3\left(\frac{1}{3}(2)^3\right)$

$= -2\left(\frac{1}{4}(-2)^4\right) + 3\left(\frac{1}{3}(-2)^3\right) + 8 - 8$

$= -8 - 8$

$= -16$