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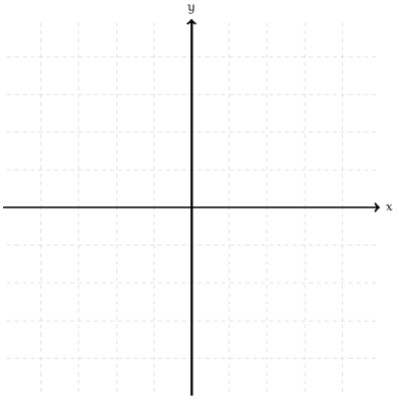
Lesson 5.3: Concavity, Points of Inflection & The Second Derivative Test

Concavity & Points of Inflection



- A graph is _____ when the tangent lines are below the graph. The slopes of the tangent lines are increasing which means:

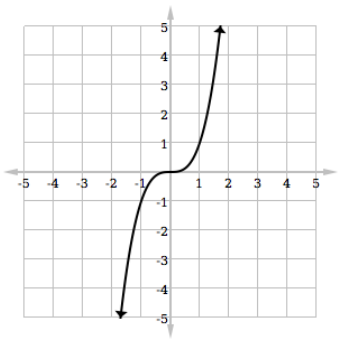
- A graph is _____ when the tangent lines lie above the graph. The slopes of the tangent lines are decreasing which means:

- A graph has an _____ if:

  1. The graph has a tangent line at the point.
  2. The graph changes concavity at that point.

Example:

1. The graph $y = x^3$ is shown at right. Find its point of inflection.



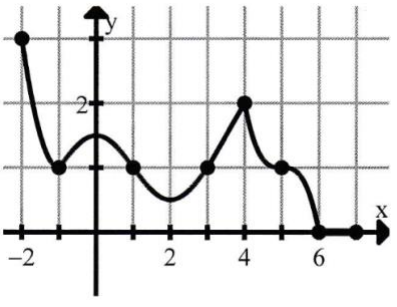
2. Use the graph at right to answer the following questions.

   a. On which intervals is the graph concave upward?

   b. On which intervals is the graph concave downward?

   c. On which intervals does the graph have no concavity?

   d. What are the points of inflection?



Finding Points of Inflection

Analytically, we find concavity intervals and points of inflection by using a _____ number line.

The procedure is parallel to the procedure used in the last lesson to find increasing/decreasing intervals and relative extrema by using a first derivative number line.

Examples:

1. Determine the points of inflection and discuss the concavity for the graph of $f(x) = x^4 + x^3 - 3x^2 + 1$.

2. If $f(x) = \frac{x^2+1}{x^2-4}$ find the intervals where the graph of f is concave upward, downward, and list the points of inflection.

The Second Derivative Test

This test does not require a second derivative number line. It does not find points of inflection. It is used to find relative extrema (max/min).

Procedure:

1. Use f' to find _____ .

2. Plug critical numbers into f'' and analyze concavity to determine if the function has a relative minimum or maximum.

Note: The Second Derivative Test does not always give an answer (when $f''(x) = 0$). Use it only when the directions require it or when the given information requires it.

Examples:

1. Use the Second Derivative Test to find the relative minimum and relative maximum points for the graph of $f(x) = -3x^4 + 6x^2$.

2. $g(x)$ is a function such that $g'(-3) = 0, g'(-1) = 0, g'(0) = 0, and\ g'(2) = 0$. If $g''(-3) = 4, g''(-1) = -2, g''(0) = 0, g''(1) = 5, and\ g''(2) = 3$, find the x-values of the relative maximum and relative minimum values when possible.