LESSON 5.3: THE FUNDAMENTAL THEOREM OF CALCULUS (PART 1)

WARM UP

Integrate.

1. $\int x^2\,dx$

$= \frac{1}{3}x^3 + C$

2. $\int_2^4 x^2\,dx$

$= \int_0^4 x^2 dx - \int_0^2 x^2 dx$

$= \frac{1}{3}(4)^3 - \frac{1}{3}(2)^3$

$= \frac{64}{3} - \frac{8}{3}$

$= \frac{56}{3}$

THE FUNDAMENTAL THEOREM OF calculus

Assume that $f(x)$ is continuous $[a,b]$. If $F(x)$ is an antiderivative of $f(x)$ on $[a,b]$, then:

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Examples:

1. Calculate the area under the graph of $f(x) = x^3$ over $[2,4]$.

$\int_2^4 x^3 dx = \left[\frac{1}{4}x^4\right]\Big|_2^4$

$= \frac{1}{4}(4)^4 - \frac{1}{4}(2)^4$

$= 64 - 4$

$= 60$

2. Find the area under $g(x) = x^{-\frac{3}{4}} + 3x^{\frac{5}{3}}$ over $[1,3]$.

$\int_1^3 x^{-3/4} + 3x^{5/3} dx = \left[4x^{1/4} + \frac{9}{8}x^{8/3}\right]\Big|_1^3$

$= \left[4(3)^{1/4} + \frac{9}{8}(3)^{8/3}\right] - \left[4(1)^{1/4} + \frac{9}{8}(1)^{8/3}\right]$

$\approx 26.325 - 5.125$

$= 21.2$

3. Calculate $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec^2(x)\,dx$

$\int_{-\pi/4}^{\pi/4} \sec^2(x) dx$

$= [\tan(x)]\Big|_{-\pi/4}^{\pi/4}$

$= \tan(\frac{\pi}{4}) - \tan(-\pi/4)$

$= 1 - (-1)$

$= 2$

4. Evaluate $\int_0^{\pi} \sin(x)\,dx$

$\int_0^{\pi} \sin(x) dx = [-\cos(x)]\Big|_0^{\pi}$

$= -\cos(\pi) - (-\cos(0))$

$= -(-1) + 1$

$= 2$

5. Evaluate $\int_0^{2\pi} \sin(x)\,dx$

$\int_0^{2\pi} \sin(x) dx = [-\cos(x)]\Big|_0^{2\pi}$

$= -\cos(2\pi) - (-\cos(0))$

$= -1 + 1$

$= 0$

6. Evaluate $\int_{-1}^{4} (4 - t)\,dt$

$\int_{-1}^4 (4-t) dt = \left[4t - \frac{1}{2}t^2\right]\Big|_{-1}^4$

$= \left[4(4) - \frac{1}{2}(4)^2\right] - \left[4(-1) - \frac{1}{2}(-1)^2\right]$

$= 16 - 8 - (-4 - \frac{1}{2})$

$= 8 - (-\frac{9}{2})$

$= \frac{25}{2}$