

## Lesson 5.4: Curve Sketching with Extrema and Points of Inflection

### Curve Sketching Recipe

1. Give the domain.
2. Reduce  $f(x)$ .
3. Find vertical asymptotes and holes.
4. Give x- and y-intercepts.
5. Find the end behavior (horizontal asymptotes or other).
6. Check for symmetry. (Optional)
7. Find increasing/decreasing intervals and relative extrema (show an  $f'$  number line).
8. Find concavity and points of inflection (show an  $f''$  number line).
9. Graph!

### Examples:

#### 1. A Rational Function

$$f(x) = \frac{3x-2}{x^2-2x+1} \quad f'(x) = \frac{-3x+1}{(x-1)^3} \quad f''(x) = \frac{6x}{(x-1)^4}$$

Domain:  $\{x \in \mathbb{R} : x \neq 1\}$

V.A.:  $x = 1$

Holes: None

x-intercepts:  $(\frac{2}{3}, 0)$

y-intercept:  $(0, -2)$

End Behavior:  $x \rightarrow \pm\infty, f(x) \rightarrow 0$

Symmetry: None

Relative Max: None

Relative Min: @  $\frac{1}{3}$  Value:  $-\frac{5}{4}$

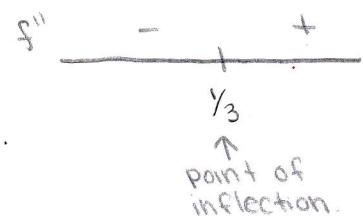
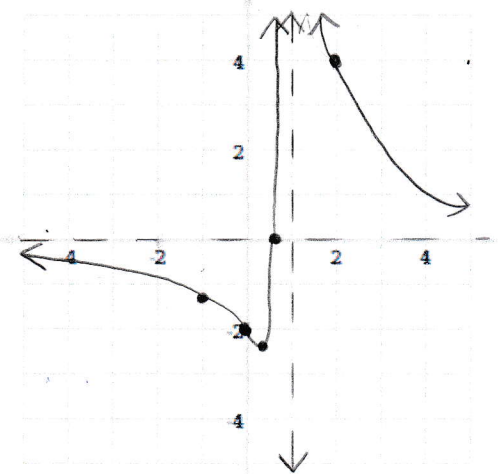
Points of Inflection:  $(\frac{1}{3}, -\frac{5}{4})$

$$f(x) = \frac{3x-2}{(x-1)^2}$$

critical #s:  $\frac{1}{3}$

$f''(\frac{1}{3}) > 0$   
 $\Rightarrow$  Relative Min @  $\frac{1}{3}$

$$f(\frac{1}{3}) = -\frac{5}{4}$$



## 2. A Radical Function

$$f(x) = \frac{x}{\sqrt{x^2+2}} \quad f'(x) = \frac{2}{\sqrt{(x^2+2)^3}} \quad f''(x) = \frac{-6x}{\sqrt{(x^2+2)^5}}$$

Domain:  $x \in \mathbb{R}$

V.A.: None

Holes: None

x-intercepts:  $(0, 0)$

y-intercept:  $(0, 0)$

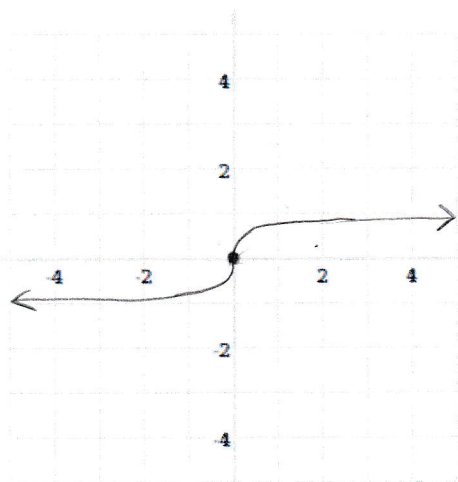
End Behavior:  $x \rightarrow \infty, y \rightarrow 1$  &  $x \rightarrow -\infty, y \rightarrow -1$

Symmetry:  $f(-x) = -f(x)$  (symmetry About the origin)

Relative Max: None

Relative Min: None

Points of Inflection:  $(0, 0)$



Critical #: 0

$$f''(0) = 0$$

