LESSON 5.4: THE FUNDAMENTAL THEOREM OF CALCULUS (PART 2)

<table>
<tr><td rowspan="2">AREA AS A function OF X</td><td>The area of a function of f with lower limit a :
$A(x) = \int_a^x f(t)\,dt =$ signed area from a to x</td></tr>
<tr><td>Example: Find a formula for the area function $A(x) = \int_3^x t^2\,dt$.
$A(x) = \int_3^x t^2\,dt = \left[\frac{1}{3}t^3\right]\Big|_3^x$
$= \frac{1}{3}(x)^3 - \frac{1}{3}(3)^3$
$= \frac{1}{3}x^3 - 9$</td></tr>
<tr><td rowspan="2">FUNDAMENTAL THEOREM OF calculus PART 2</td><td>Assume that f(x) is continuous on an open interval I and let $a \in I$. Then, $A(x) = \int_a^x f(t)\,dt$ is an antiderivative of f(x) on I. ($A'(x) = f(x)$)
$A'(x) = \frac{d}{dx}\int_a^x f(t)\,dt = f(x)$
*A(x) satisfies the initial condition $A(a) = 0$</td></tr>
<tr><td>Examples:
1. Let $F(x)$ be the particular antiderivative of $f(x) = \sin(x^2)$ satisfying $F(-\sqrt{\pi}) = 0$. Express $F(x)$ as an integral.
$F(x) = \int_{-\sqrt{\pi}}^x \sin(t^2)\,dt$
2. Find the derivative of $A(x) = \int_2^x \sqrt{1+t^3}\,dt$ and calculate $A'(2)$, $A'(3)$ and $A(2)$.
$A'(x) = \frac{d}{dx}\int_2^x \sqrt{1+t^3}\,dt = \sqrt{1+x^3}$
$A'(2) = \sqrt{1+2^3} = 3$ $A'(3) = \sqrt{1+3^3} = \sqrt{28}$ $A(2) = 0$
3. Find the derivative of $G(x) = A(x^2) = \int_{-2}^{x^2} \sin(t)\,dt$.
$G'(x) = (A'(x^2))(2x) = 2x\sin(x^2)$</td></tr>
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