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Lesson 6.1: Antiderivatives

Warm-Up: Differentiate each of the following.

1. $f(x) = x^3$
2. $f(x) = x^3 - 10$
3. $f(x) = x^3 + c$ (where c is any number)

What do you get when you antidifferentiate (integrate) $f'(x) = 3x^2$?

$$f(x) = \_\_\_\_\_\_\_\_$$

The symbol $\int$ is called an **integral symbol** and tells you to integrate (antidifferentiate) the expression which follows it.

That expression is called an **integrand**. dx indicates that you are integrating with respect to the variable x, but does not affect the integration process.

C is called the **constant of integration** and must be written as part of your answer when you are anti-differentiating.

Integration Rules	
Power Rule	$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, \quad where\ n \neq -1$
Constant Rule	$\int k\,dx = kx + C, \quad where\ k\ is\ any\ constant$
Scalar Multiple Rule	$\int k\,f(x)\,dx = k\int f(x)\,dx, \quad where\ k\ is\ any\ constant$
Sum Rule	$\int \big(f(x) + g(x)\big)\,dx = \int f(x)\,dx + \int g(x)\,dx$
Trig Rules	$\int \cos(x)\ dx = \sin(x) + C$ $\int \sin(x)\ dx = -\cos(x) + C$ $\int sec^2(x)\,dx = \tan(x) + C$ $\int csc^2(x)\,dx = -\cot(x) + C$ $\int \sec(x)\tan(x)\ dx = \sec(x) + C$ $\int \csc(x)\cot(x)\ dx = -\csc(x) + C$

A Few Notes About Integrals:

1. Constants can be "factored out" of the integral expression (in reference to the scalar multiple rule), but **NEVER** factor out a variable.
2. Sometimes, an initial condition can be given that makes it possible to solve for C.
3. Position → Velocity → Acceleration (Differentiate)
   Acceleration → Velocity → Position (Integrate)

Examples:

1. Evaluate. (Integrate)

   a. $\int x^3\, dx$

   e. $\int (2y^2 + 4y + 1)\, dy$

   b. $\int 2\, dx$

   f. $\int \left(\frac{3}{x^2} - \frac{1}{\sqrt{x}}\right) dx$

   c. $\int (t^4 + 2)\, dt$

   g. $\int \frac{\sqrt{x}+1}{x^2}\, dx$

   d. $\int (2\cos(\theta) - 3\sin(\theta))\, d\theta$

   h. $\int \frac{\cos(x)}{sin^2(x)}\, dx$

2. If $f'(x) = x^{-3}$ and $f(1) = \frac{3}{2}$, find $f(x)$.

3. The acceleration of a particle at time t is given by $a(t) = 4t - 3$, $v(1) = 6$ and $s(2) = 5$.

   a. Find the velocity equation: $v(t)$.

   b. Find the position equation: $p(t)$.

4. Given that on Earth, the acceleration of an object due to gravity is approximately $-32\ ft/sec^2$ (negative indicates downward), develop:

   a. The equation for velocity of the object. ($v_0 = initial\ velocity$)

      $v(t) =$

   b. The equation for the position of the object. ($s_0 = initial\ position$)

      $s(t) =$

Note: The two equations $v(t) = -32t + v_0$ and $s(t) = -16t^2 + v_0 t + s_0$ may be used for any motion affected only by Earth's gravity.